Solution Manual

Problem #1

a) $2\begin{bmatrix}5\\2\\3\end{bmatrix} - \begin{bmatrix}10\\6\\4\end{bmatrix} + 2\begin{bmatrix}1\\1\\0\end{bmatrix} - 2\begin{bmatrix}2\\1\\1\end{bmatrix} = 2v_3 - v_4 + 2v_2 - 2v_1 = 0$

non-trivial linear combination vanishes so they cannot be a basis

b) Systematically we have to find dim(E) by Gaussian elimination

$$\begin{array}{cccc} \begin{bmatrix} 2 & 1 & 5 & 10 \\ 1 & 1 & 2 & 6 \\ 1 & 0 & 3 & 4 \end{bmatrix} \\ v_1 \ v_2 \ v_3 \ v_4 \end{array} \xrightarrow[\substack{v_4 - 10v_2 \\ v_1 - 2v_2}]{v_3 - 5v_2} \begin{array}{c} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 1 & -3 & -4 \\ 1 & 0 & 3 & 4 \end{bmatrix} \\ c_1 \ c_2 \ c_3 \ c_4 \end{array} \xrightarrow[c_2 \to c_1]{c_1 \to c_2} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & -3 & -4 \\ 0 & 1 & 3 & 4 \end{bmatrix} \\ c_1 \ c_2 \ c_3 \ c_4 \end{array} \xrightarrow[\substack{c_4 + 4c_2 \\ c_1 + c_2}]{c_3 + 3c_1}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \Longrightarrow$$ So the dim of span of Columns is 2.

c) from computation above E is generated by two vectors $e_1 = (1,0,1)$, $e_2 = (0,-1,1)$. in order to find A such that $A(E)=0$, it's sufficient to have $Ae_1 = 0$ and $Ae_2 = 0$

take $A = \left[c_1 \,\vdots\, c_2 \,\vdots\, c_3 \right]$, $A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = [c_1] + [c_3] = 0 \qquad [c_1] = [c_3]$

$$A \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = 0 \Rightarrow -[c_2] + [c_3] = 0 \Rightarrow [c_3] = -[c_2]$$

So every A of the form $\begin{bmatrix} a & -a & a \\ b & -b & b \\ c & -c & c \end{bmatrix}$ for arbitrary a, b, c

works.

d) from the process of Gaussian elimination we have shown

that $\text{Span}(v_1, v_2) = \text{Span}(e_1, e_2)$ so we have to find w

such that to be linear independent of $\text{span}(e_1, e_2)$

$\text{span}(e_1, e_2) = \begin{bmatrix} t \\ -k \\ t+k \end{bmatrix}$ for $t, k \in \mathbb{C}$ so obviously $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ is not

in this space so is independent

Problem 2 ; (a) Let $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$$AB = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \neq BA = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$

(b) $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$$B_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$