Math 108 – Calculus I Practice Final 12/08/2010

▶ **PRINTED name:** _____

▶ **Please circle your section:**

(1)	T 1:30	Gilman 17	Ariturk, Sinan
(2)	T 3:00	Hodson 210	Tran, Timothy
(3)	Th 1:30	Maryland 309	Ravit, Jason
(4)	Th 3:00	Hodson 316	Tran, Timothy

▶ **Write out and SIGN the pledge:**

I attest that I have completed this exam without unauthorized assistance from any person, materials, or device.

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Signature: Date:

Grading	
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Total:	

▶ This is a 3-hour closed book exam. No notes, books, or calculators are allowed.

▶ This examination booklet contains 8 problems, on 14 sheets of paper including the front cover. Some questions take more than one page. Please detach the last two pages, which are intended for use as scrap paper.

▶ Show all work. The correct answer is worth no points without any argumentation. Feel free to use the other side of a paper if necessary, but make sure to give directions to match your solution and the problem.

1 (40 pts, 5 for each) Evaluate the following limits

a) $\lim_{x\to -3} \frac{x^2+5x+6}{x^2+4x+3}$

b) $\lim_{x\to \infty} \frac{x^2+5x+6}{x^2+4x+3}$

c) $\lim_{x\to 0} \frac{x^2+5x+6}{x^2+4x+3}$

d) $\lim_{x\to \infty} (\sqrt{x^2+4} - x)$

e) $\lim_{x\to 0} e^{x\cos(e^{-1/x})}$

f) $\lim_{x\to\infty} \frac{1}{x\sin(2/x)}$

g) $\lim_{x\to\infty}(1+\frac{1}{2x})^x$

h) $\lim_{x\to\infty} x^{1/x}$

2 (30 pts, 6 for each.) Find the derivatives $y' = f'(x)$ of the following functions $y = f(x)$.

a) $f(x) = \ln(\sin(x^2 + 1))$

b) $f(x) = (x^2 + 1)\tan^{-1}(x)$

c) $f(x) = (\cos x)^{\sin x}$ $(x \in (0, \pi/2))$

d) $\int_{\ln x}^{x} \sin(e^t)dt$

e) $xy + \ln y = 2x^2 + y^2 + 3$

3 (40 pts, 10 for each.) Evaluate the following integrals.

a) $\int (\sec\theta)^2 \tan\theta d\theta$

b) $\int_{-1}^{1} \frac{x^3}{1+x^4} dx$

c) $\int_0^{-1} \frac{2x}{1+x^4} dx$

d) $\int_{-2}^{2} |x^2 - 1| dx$

4 (10 pts.) Prove that there is one and only one real root for the equation $2x - 1 - \sin x = 0$.

5 (8+8+4=20 pts.) Let $f(x) = e^{2x} + e^x$.

i) Show that $f(x)$ is one to one.

ii) Find the inverse function $f^{-1}(x)$ (state domain explicitly).

iii) What is the range of $f^{-1}(x)$?

6 (20 pts.) Let $g(x) = \int_0^{x^3} \cos t dt$,

(a) Find the maximum possible value of $\frac{g'(x)}{x^2}$ for $x > 0$.

(b) Find the absolute maximum value of $g(x)$ in the interval $[0, (\frac{\pi}{2})^{1/3}]$.

7 (20 pts.) Sketch the regions and find the areas.

a) between $y = \cos x$ and $y = 2 - \cos x$, for $0 \leq x \leq 2\pi$.

b) the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.

8 (20 pts.) Find the two volumes generated by rotating the regions bounded by the given curves about the specified axes. Sketch the regions.

(a) $y = 3 + 2x - x^2$ and $x + y = 3$, about y-axis,

(b) $y = x^2$ and $x = y^2$, about $y = -1$.

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Some Formulas

Four your convenience, please DETACH this page before the Exam

$$a^2 - b^2 = (a-b)(a+b),\ a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

Binomial formula for $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric, logarithmic and exponential functions

$\sin^2 x + \cos^2 x = 1$	$\sin(x+y) = \sin x \cos y + \sin y \cos x$
$\cos(x+y) = \cos x \cos y - \sin x \sin y$	$\sec^2 x = 1 + \tan^2 x$
$\log_a b = \frac{\ln b}{\ln a}$	$\ln a = b \Leftrightarrow e^b = a$
$e^{a+b} = e^a \cdot e^b$	$(e^a)^b = e^{ab}$
$\ln(a \cdot b) = \ln a + \ln b$	$\ln(a^b) = b \ln a,\ a^b = e^{b \ln a}$

Inverse trigonometric functions

$f(x)$	domain	range
$\sin^{-1} x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$