Math 108 Final Exam — Dec. 9, 2015

Name: ______________________________________

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Section Time (Circle one): T 1:30-2:20 T 3:00-3:50 Th 1:30-2:20 Th 4:30-5:20 Th 3:00-3:50

- Complete the following problems. In order to receive full credit, please make sure to *justify your answers*. You are free to use results from class or the course textbook as long as you clearly state what you are citing.
- **You have 3 hours** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted (nor are they needed). If you finish early, you must hand your exam paper to a proctor.
- Please check that your copy of this exam contains 10 numbered pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

The following boxes are strictly for grading purposes. Please do not mark.

Question	Points	Score
1	40	
2	10	
3	20	
4	10	
5	20	
6	10	
7	40	
8	10	
Total	160	

1. Compute the following limits. You may use any technique you like as long as you justify your reasoning.

   (a) (10 points) $\lim_{x\to 4} \log_2\left(\frac{\sqrt{x}-2}{x-4}\right)$.

   (b) (10 points) $\lim_{x\to 1^+}(2f(x)-x)$, where f satisfies $|f(x)-2| < \frac{1}{2}|x-1|$ for all $x \in (1,2)$.

(c) (10 points) $\lim_{x\to\infty} x^{1/x}$.

(d) (10 points) $\lim_{x\to 1} \frac{G(x)}{x-1}$, where $G(x) = \int_1^{x^2} f(t)\,dt$ and f is a continuous function with $f(1) = -1$.

2. (10 points) Determine values of c and d so that the function $f(x) = \begin{cases} 2x-1 & x > c \\ d+1 & x = c \\ 3x+2 & x < c. \end{cases}$ is continuous.

3. Determine the following values by differentiating.

   (a) (10 points) Value of $f'(3)$, where $f(x) = xg(g(x))$, $g(3) = 3$ and $g'(3) = -1$.

   (b) (10 points) Value of $\frac{dy}{dx}$ at $(2, 0)$, where $\frac{dy}{dx}$ is the slope of the tangent line to the curve $x^3+2xy = 8$.

4. Let f be a function defined on $(0, 9)$ with the property that $f(5) = 2$, $f'(5) = -1$ and $f''(x) < 0$ for all $x \in (0, 9)$.

   (a) (5 points) Determine $L(x)$, the linearization of f at $x = 5$, and use it to approximate $f(5.01)$.

   (b) (5 points) Determine whether the approximate value found in part a) is an overestimate or underestimate of $f(5.01)$ or if there is not enough information to tell. Remember to justify your reasoning.

5. (20 points) Determine the extreme values of $f(x) = x + \frac{4}{x}$ on the interval $[1, 4]$.

6. (10 points) Suppose f is a continuous function that is increasing on $[-1, 5]$ and satisfies $f(-1) = -2$ and $f(5) = 1$. Based on this information, determine upper and lower bounds for $\int_{-1}^{5} f(t)\,dt$.

7. Compute the following definite and indefinite integrals.

   (a) (10 points) $\int_0^2 |x^2 - x| \, dx$.

   (b) (10 points) $\int (t-1)\sin(t^2 - 2t) \, dt$.

(c) (10 points) $\int_0^2 2f(x) - 1\,dx$, given that $\int_0^3 f(t)\,dt = -3$ and $\int_2^3 f(x)\,dx = 2$.

(d) (10 points) $\int_0^1 \frac{F'(\arctan(s))}{1+s^2}\,ds$, given that $F(0) = 0$ and $F(\frac{\pi}{4}) = -1$ and F' is continuous.

8. (10 points) Let R denote the region inside the ellipse $x^2 + 4y^2 = 16$ and above the line $y = \frac{1}{2}x - 2$. Express the area of R as an integral (you do not need to evaluate it).

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