

**110.108 CALCULUS 1**  
**FALL 2012**  
**MIDTERM 1**

Name: \_\_\_\_\_

Recitation section:

- 1. Tues 1:30 (P. Shao)
- 2. Tues 3:00 (P. Shao)
- 3. Thurs 4:30 (B. Elder)
- 4. Thurs 3:00 (Q. Giang)
- 5. Thurs 1:30 (Q. Giang)

Work quickly and carefully, and write your solutions clearly. Please show your work; partial credit will be given generously.

*Statement of ethics*

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: \_\_\_\_\_ Date: \_\_\_\_\_

Problem	Score
1	/12
2	/10
3	/10
4	/10
5	/10
TOTAL	/52

**Problem 1** (12 points). Compute the following limits. Briefly justify the steps you take.

(a)  $\lim_{x \rightarrow 0} e^{\sin(\frac{\pi}{2}(x^2-x+1))}$

(b)  $\lim_{x \rightarrow 9} \frac{9-x}{\sqrt{x}-3}$

(c)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

**Problem 2.** [10 points]

(a) Let  $f$  be a function defined on an open interval containing  $a$ . Define “ $f$  is continuous at  $a$ .”

(b) Let  $f(x) = \begin{cases} (cx + 1)^2, & x \leq 1 \\ 4 - c^2x^2, & x > 1 \end{cases}$ .

Find a value of  $c$  such that  $f$  is continuous on all of  $\mathbb{R}$ .

**Problem 3.** [10 points] Give an  $\epsilon$ - $\delta$  proof that  $\lim_{x \rightarrow 2} \frac{3x^2 - 15x + 18}{2x - 4} = -\frac{3}{2}$ .

**Problem 4.** [10 points]

(a) Let  $f(x) = \sqrt{x-2}$ . Using the definition of derivative, compute  $f'(6)$ .

(b) Let

$$f(x) = \frac{4^x}{\sin(2x) + \sec(2x)}.$$

Using various differentiation rules from class, compute  $f'(x)$ . You do not need to simplify your answer.

**Problem 5.** [10 points] Let  $f(x) = \frac{1}{4}x^2 + 1$ . Find an equation for the tangent line to the graph of  $f$  at the point  $(a, f(a))$ . For which  $a$  does this tangent line pass through the origin  $(0, 0)$ ?