

Mathematic 108, Summer 2019: Assignment #1

Due: Tuesday, July 9th

Instructions: Please ensure your name appears on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Determine the (largest) domain and range of the function given by $f(x) = \frac{x}{\sqrt{4-x^2}}$. Sketch the graph of this function.

Problem #2. Determine the (largest) domain of the function given by $f(x) = \frac{1}{1+\tan^2(x)}$. Explain why this is *not* the same function as $g(x) = \cos^2(x)$.

Problem #3. Ann leaves Baltimore at 6:00AM and drives at a constant speed south along I-95. She passes Washington, DC, which is 40 mi from Baltimore, at 7:30AM

- Express the distance traveled (in miles) in terms of the time traveled (in hours).
- Express the distance traveled (in kilometers) in terms of the time of day (in hours).
- How are these two functions related?

Problem #4. Let $f(x) = \frac{1}{x}$ and

$$g(x) = \begin{cases} 0 & 1 \leq x \leq 2 \\ -\frac{1}{x} & x > 2. \end{cases}$$

Determine the formulas for the following functions and their domains:

- $f + g$.
- $f \circ g$.
- $g \circ f$.

Problem #5. Express the following functions in the form $f \circ g$ where f is a rational function and g is a trigonometric function:

- $u(t) = \frac{\cos^2(t)}{1-\cos(t)}$.
- $w(t) = \frac{\cos(t)}{\sin^4(t)}$.

Problem #6. Determine the largest value L so that the $f(x) = (x-2)^2 + 2$ is one-to-one on the interval $(-L, L)$. Find the formula for f^{-1} and its domain.

Problem #7. Find a formula for f^{-1} and determine its domain when $f(x) = 1 + \sqrt{1-2x}$.

Problem #8. Simplify $\cos(2 \arccos(2x))$.

Problem #9. Evaluate the limit, if it exists.

- $\lim_{x \rightarrow -1} \frac{x+1}{x^3+1}$.
- $\lim_{x \rightarrow 1} (x^2 - 1)(x^2 + 1)$.
- $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{|x|}$.

Problem #10. Determine the following infinite limits

- $\lim_{x \rightarrow 0^+} \ln\left(\frac{1}{x}\right)$.
- $\lim_{x \rightarrow 0^+} \left(\frac{1}{\sqrt{x}} - \ln(x)\right)$.
- $\lim_{x \rightarrow \left(-\frac{\pi}{2}\right)^-} x \tan(x)$

Problem #11. Let $f(x) = \begin{cases} 2 - x^2 - c & x < -1 \\ \sqrt{x+c} & x > -1 \\ 10 & x = -1. \end{cases}$ Determine values c so that $\lim_{x \rightarrow -1} f(x)$ exists.

Problem #12. Suppose that $f(x)$ is defined near $x = -1$ and satisfies $-x^4 - x^2 + 4 \leq f(x) \leq 4 + 2x^2$. Calculate $\lim_{x \rightarrow -1} f(x)$.

Problem #13. Use the ϵ, δ definition of limit to show that $\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$.

Problem #14. Using limit laws, show that the following functions are continuous at the given value a .

- a) $f(x) = \frac{x^3 - 1}{x + 1}$, $a = 1$.
- b) $f(x) = \frac{\sqrt{x^2 + 4} + 2}{x + 2}$, $a = 0$.

Problem #15. Use continuity to evaluate the following limits.

- a) $\lim_{x \rightarrow 0} \tan(x^2 - x)$.
- b) $\lim_{x \rightarrow 1} \ln\left(\frac{3-x}{x^2+1}\right)$

Problem #16. Explain why the function is discontinuous at the given a and determine, if possible, the type of the discontinuity.

- a) $a = -\frac{1}{2}$ and $f(x) = \frac{2x-1}{(4x+2)^2}$.
- b) $a = -1$ and $f(x) = \begin{cases} \frac{x+1}{1-\sqrt{-x}} & x < 0, x \neq -1 \\ 0 & x = -1. \end{cases}$
- c) $a = 0$ and $f(x) = \begin{cases} \cos(x) & x < 0 \\ -\cos(x) & x \geq 0. \end{cases}$

Problem #17. Let f be continuous on $[2, 5]$. If f is zero only at $x = 5$ and $f(3) = -3$, then can $f(4) = 2$?

Problem #18. Determine value(s) c so that $f(x) = \begin{cases} -\frac{9}{x^2+c} & -3 \leq x \leq 2 \\ 2x - c & 2 < x < 5. \end{cases}$ is continuous on $(-3, 5)$.

Problem #19. Show that the function $f(x) = \begin{cases} x \sin\left(\frac{1}{x^2}\right) & x \neq 0 \\ 0 & x = 0. \end{cases}$ is continuous on $(-\infty, \infty)$.

Problem #20. Find the limit or explain why it doesn't exist.

- a) $\lim_{x \rightarrow -\infty} \frac{3x^3 + x^2 - x + 1}{x^3 - 1}$
- b) $\lim_{x \rightarrow 0^-} \tan^{-1}\left(\frac{1}{x}\right)$

Suggested Book Problems (not to be handed in).

- a) Section 1.1: # 4, # 14
- b) Section 1.2: # 10
- c) Section 1.3: # 4, # 32, # 34
- d) Section 1.4: # 20
- e) Section 1.5: # 10, # 30, # 56.
- f) Section 2.1: # 2
- g) Section 2.2: # 4, # 16
- h) Section 2.3: #2, #6, # 10, # 14, # 50.
- i) Section 2.4: #2, #38
- j) Section 2.5: #4, #24, # 56
- k) Section 2.6: #42, #58