# Mathematic 108, Summer 2019: Assignment #2

Due: Tuesday, July 16th

*Instructions:* Please ensure your name appears on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

**Problem #1.** Consider the function $f(x) = \begin{cases} 2(x-1)^2 - 1 & x \leq 1 \\ x^3 & 2 \leq x < 3. \end{cases}$

a) Determine the equation of the secant line between $(0, f(0))$ and $(2, f(2))$.
b) Let $m(x)$ be function whose value at $x$ is the slope of the secant line between $(0, f(0))$ and $(x, f(x))$. Determine $m(x)$ and its domain.
c) Calculate $\lim_{x \to 0} m(x)$.

**Problem #2.** Calculate $f'(0)$ for $f(x) = \begin{cases} 3 + x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 3 & x = 0. \end{cases}$

**Problem #3.** Let $g(x) = x^{2/3}$.

a) Show that $g'(0)$ doesn't exist.
b) Calculate $g'(a)$ for $a \neq 0$.

**Problem #4.** Find the equation of the tangent line and of the normal line to the curve at the given point.

a) $y = 2x - x^2 + e^x$ at $(0, 1)$.
b) $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ at $(1, 0)$.

**Problem #5.** Find constants $a, b$ so that $f(x) = x^2 + ax + b$ is tangent to the line $y = 2x - 3$ at $(2, 1)$.

**Problem #6.** Let $g(x) = xe^x$. Compute all $g^{(n)}(x)$ where $n$ is a positive integer.

**Problem #7.** Suppose $f(x)$ is a differentiable function. Determine an expression for the derivative of the following functions

a) $g(x) = \frac{f(x)}{x}$.
b) $g(x) = \frac{1+f(x)}{1-f(x)}$.
c) $g(x) = x(f(x))^2$.

**Problem #8.** Suppose that $g$ is differentiable.

a) Use the Quotient Rule to show $\frac{d}{dx}\left(\frac{1}{g(x)}\right) = -\frac{g'(x)}{g(x)^2}$.
b) Use the Chain Rule and the fact that $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$ to show the same thing.

**Problem #9.** Determine the constants $m$ and $b$ so that the function $f(x) = \begin{cases} xe^{1-x^2} & x \leq 1 \\ mx + b & x > 1 \end{cases}$ is differentiable on $(-\infty, \infty)$.

**Problem #10.** Let $h(x) = \sqrt{25 - 3f(x)}$ were $f(1) = 3$ and $f'(1) = 4$ determine $h'(1)$.

**Problem #11.** Determine the value(s) of $\alpha$ and $\beta$ so that $f(x) = e^{\alpha x} \cos(\beta x)$ satisfies $f''(x) - 2f'(x) + 2f(x) = 0$ for all $x$. That is, so $f$ is a solution to the differential equation $y'' - 2y' + 2y = 0$.

**Problem #12.** Compute the following limits

a) $\lim_{x\to 0} \frac{\sin(x)}{\sin(\pi x)}$.
b) $\lim_{x\to 0} \frac{\sin(x^2)}{x}$.
c) $\lim_{x\to 0} \frac{\cos(x)-1}{\sin(x)}$.

**Problem #13.** Find $\frac{dy}{dx}$ by implicit differentiation

a) $xe^{-y} = x^2 - y$.
b) $x\cos(y) + y = -1$.

**Problem #14.** Determine the equation of the tangent line to the curve $\cos(y) + \sin(y) = x$ at $(-1, 3\pi)$.

**Problem #15.** Compute $y'' = \frac{d^2y}{dx^2}$ by implicit differentiation when $x^2 - y^2 = 1$.

**Problem #16.** Suppose $f(x)$ satisfies $f(f(x)) = \frac{1}{2}\left(f(x)^2 + x^4\right)$ and $f(1) = 1$. What are the possible values of $f'(1)$?

**Problem #17.** Suppose $x > 0$ and $x^y = y$.

a) Compute $\frac{dy}{dx}$.
b) Determine the tangent at $(1, 1)$.

**Problem #18.** Differentiate the following functions

a) $f(x) = \sqrt{1 + (\ln(x))^2}$.
b) $f(x) = \cos(\ln|x|)$.
c) $f(x) = (\sqrt{x})^{x-1}$.

**Problem #19.** An ant is crawling along a hyperbola $x^2 - y^2 = 3$. As it reaches the point $(2, -1)$ its $x$ coordinate increases at a rate of 5. Determine the rate of change of the $y$ coordinate when this occurs.

**Problem #20.** Linearize the given function at the given value and use it approximate the given number.

a) Linearize $f(x) = x^4$ at $a = 2$. Approximate $f(2.02)$.
b) Linearize $f(x) = \sqrt{x}$ at $a = 36$. Approximate $f(36.1)$.

**Suggested Book Problems (not to be handed in).**

a) Section 2.7: #8 , # 38
b) Section 2.8: #22, #26, #42
c) Section 3.1: # 56, # 72
d) Section 3.2: #18, #46, #50
e) Section 3.3: # 12, # 18
f) Section 3.4: # 2, #36, # 64
g) Section 3.5: #56
h) Section 3.6: #28, #34
i) Section 3.9: #18, #20, #22, #28
j) Section 3.10: #4, #14, #30