Mathematic 108, Summer 2019: Assignment #3

Due: Tuesday, July 23rd

Instructions: Please ensure your name appears on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Determine the critical numbers of the following functions

a) $f(x) = 2x^3 + x^2 + 2x - 4$.
b) $f(x) = |x - 1| + x^2$.

Problem #2. Find the absolute maximum and minimum values of the given functions on the given intervals

a) $f(x) = -1 + 36x - 3x^3$, $[-3, 1]$.
b) $f(x) = e^x - x$, $-1 \leq x \leq 1$.
c) $f(x) = x + \cos(x)$, $[0, 2\pi]$.

Problem #3. Explain why the function $f(x) = e^{-2x} - x^{101} - 2$ has no local maxima or minima.

Problem #4. Let $f(x) = 1 - x^{4/5}$. Show that $f(-1) = f(1)$, but there is no value c in $(-1, 1)$ so that $f'(c) = 0$. Why does this not contradict Rolle's theorem.

Problem #5. Show that the equation $e^{2x} + e^x = -x$ has exactly one real solution.

Problem #6. Use the Mean Value Theorem to show that for all x, y

$$|\arctan(x) - \arctan(y)| \leq |x - y|.$$

Problem #7. Determine the intervals of increase and decrease and intervals of concavity for the following functions.

a) $f(x) = x^2 e^{-x}$.
b) $f(x) = \cos^2(x) + 2\sin(x)$ and $-2\pi \leq x \leq 2\pi$.

Problem #8. For what values of c is the function $f(x) = cx + \frac{1}{x^2+3}$ decreasing on $(-\infty, \infty)$? (Hint: try to determine the maximum value of $f'(x)$).

Problem #9. Suppose f is twice differentiable on $(-1, 1)$, $f(0) = 3$, $f'(0) = -1$ and $f''(x) > 0$ on $(-1, 1)$. Based on this information, determine an approximate value of $f(0.1)$. Is this an overestimate or underestimate of the true value.

Problem #10. Give an example of a continuous function with domain $[-1, 1]$ with a local maximum, but no local minimum.

Problem #11. Give an example of a function f with continuous second derivative for which f'' is zero at some point and whose graph does not have an inflection point.

Problem #12. Determine whether the following functions have an absolute maximum value and absolute minimum value on the given domain. If it does determine the value.

a) $f(x) = \frac{1}{1+e^{-x^2}}$ on $D = (-\infty, \infty)$.
b) $f(x) = \tan^{-1}(x) + \frac{1}{1+x^2}$ on $D = (-\infty, \infty)$.
c) $f(x) = x - \sqrt{x^2 + 3}$ on $D = [0, \infty)$.

Problem #13. Let $f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$ and $g(x) = \sin(x)$.

a) Use the limit laws to show that $\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$ (Hint: Consider $\lim_{x\to 0} \frac{f(x)/x}{g(x)/x}$).
b) Determine $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ how do you reconcile this with a) and L'Hospital's Rule.

Problem #14. Use L'Hospital's Rule to evaluate the following limits

a) $\lim_{x\to 0} \frac{\arcsin(2x)}{x}$.
b) $\lim x \to 0^+ \left(\frac{1}{x} - \frac{1}{\arctan(x)}\right)$.
c) $\lim_{x\to 0^+} (1 + \sin(2x))^{1/x}$.

Problem #15. Suppose f is differentiable, $f(3) = 1$ and $f'(3) = -2$. Evaluate $\lim_{x\to 0} \frac{f(3+x)-f(3-4x)}{x}$.

Problem #16. Use the methods of Section 4.5 to sketch the following curves

a) $y = \frac{(x-2)^2}{x^2+1}$
b) $y = x - \sin(2x)$
c) $y = \sqrt{1+x^2} - x$

Problem #17. Consider the family of polynomials $P_c(x) = x^3 + 3cx^2 + 3x$.

a) Determine the values of c so that P_c has both a local maximum and a local minimum.
b) Sketch the graph of $y = P_c(x)$ for a value c for which c has both a local maximum and minimum and sketch the graph for a value c for which it does not.

Problem #18. Determine the point on the curve $y = 2\sqrt{x}$ that is closest to the point $(12, 0)$.

Problem #19. A piece of wire of length $20m$ is cut into two pieces. One piece is bent into a square and the other into a circle. How should the wire be cut so total area enclosed is

a) Maximal.
b) Minimal.

Problem #20. Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Suggested Book Problems (not to be handed in).

a) Section 4.1: #4, #38, #56
b) Section 4.2: #6, #18, #22, #26
c) Section 4.3: #8, #24, #52
d) Section 4.4: #4, #32, #44, #76, #88
e) Section 4.5: #2, #12, #50, #72
f) Section 4.6: #28
g) Section 4.7: #4, #10, #44, #48, #76