Mathematic 108, Summer 2019: Assignment #4

Due: Tuesday, July 30th

Instructions: Please ensure your name appears on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Find the most general form of the antiderivative of the given functions and check your answer by differentiating.

a) $f(x) = 2x^2 + 3x + 3$
b) $f(x) = e^x + 3x^2$
c) $f(x) = x\sqrt{x} - \frac{1}{1+x^2}$

Problem #2. Find the function, f, that satisfies:

a) $f''(t) = 2\cos(2t)$, $f(0) = 1$ and $f'(0) = 0$.
b) $f'(t) = \frac{1}{t}$ and $f(1) = 1$ and $f(-1) = 0$.

Problem #3. Determine the differentiable function f so that $f(0) = 1$ and $f'(x) = \begin{cases} 2x+1 & x < -1 \\ x & x \geq -1 \end{cases}$.

Problem #4. Determine the continuous function g so that $g(0) = 0$ and $g'(x) = \begin{cases} 1-x & x < 2 \\ \frac{4}{x^2} & x > 2 \end{cases}$. Is this function differentiable?

Problem #5. Suppose, $\int_0^1 f(y)dy = -2$, $\int_0^2 f(t)dt = -3$ and $\int_1^3 f(x)dx = 0$. Compute $\int_2^3 f(x)dx$.

Problem #6. Suppose that $|f(x)| \leq 2|x|$. Determine the largest and smallest possible values for $\int_1^3 f(x)dx$.

Problem #7. Suppose that the graph of f is concave up on $(-2, 2)$, $f(0) = 2$ and $f'(0) = 2$. Determine the smallest possible value of $\int_{-1}^1 f(t)dt$.

Problem #8. Let $f(x) = \begin{cases} -2x & x \leq 2 \\ 3 & x > 2. \end{cases}$ Compute $F(x) = \int_0^x f(t)dt$.

Problem #9. Evaluate the following definite integrals.

a) $\int_{-1}^1 x^{25} dx$
b) $\int_0^1 \frac{2}{1+x^2} dx$
c) $\int_1^2 t + t^{-1} dt$.

Problem #10. If $F(x) = \int_x^{x^2} \cos(t^2)dt$ compute $F'(x)$.

Problem #11. Compute $\int_{-\pi}^{\pi} |\sin(x)|dx$.

Problem #12. If $f(3) = 3$, f' is continuous and $\int_{-1}^3 f'(t)dt = 12$, then compute $f(-1)$.

Problem #13. Evaluate the following indefinite integrals and then check your work by differentiating

a) $\int \sqrt{t}(t^2 - t - 1)dt$.
b) $\int 1 - \tan^2(\theta) + \cos(\theta)d\theta$.
c) $\int 2^x(3^{-x} + 3^x)dx$.

Problem #14. Suppose the velocity of a particle is $v(t) = t^2 - 2t - 3$

a) Determine the total displacement of the particle from $t = 1$ to $t = 4$.
b) Determine the total distance the particle travels from $t = 1$ to $t = 4$.

Problem #15. Evaluate the following definite integrals

a) $\int_0^2 \frac{1}{3x+2} dx$.
b) $\int_{-1}^2 xe^{-x^2} dx$.
c) $\int_0^2 \frac{x}{\sqrt{1+4x}} dx$.

Problem #16. Evaluate the following indefinite integrals

a) $\int \sin(2\theta)\sqrt{1+\cos(2\theta)}d\theta$.
b) $\int \frac{x}{1+x^4} dx$.
c) $\int \frac{(\ln(x))^3}{x} dx$.

Problem #17. If $\int_1^5 f(x)dx = -3$, then determine $\int_1^3 f(2x-1)dx$.

Problem #18. Sketch the regions enclosed by the given curves and find their area

a) $y = \cos(x), y = 2 - \cos(x), 0 \leq x \leq 2\pi$.
b) $y = x^3, y = x$.

Problem #19. Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$ and the x-axis.

Problem #20. For what value c is the region between $y = c^2 - x^2$ and $y = x^2 - c^2$ equal to 576

Suggested Book Problems (not to be handed in).

a) Section 4.9: #18, #34, #38, #50, #78
b) Section 5.1: #2, #24
c) Section 5.2: #18, #34, #40, #56
d) Section 5.3: #14, #34, #48, #64