CALCULUS 110.108, FALL 2014,
PRE-FINAL PRACTICE
JOHNS HOPKINS UNIVERSITY

Problem 1. Find the domains of definition of the following functions

(a) $f(x) = \arcsin\left(\frac{x-3}{2}\right) - \log_e(4-x)$

(b) $g(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

Problem 2. Plot the graph of the following function, clearly marking the x and y intercepts, if any.

$$f(x) = 2\sin\left(3x + \frac{3\pi}{4}\right).$$

Is the function periodic, if so what is the period?

Problem 3. Find the derivative of the following functions

(a) $\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$

(b) $\arcsin^2\left(\log_e(a^3 + x^3)\right)$

(c) $\sqrt[3]{\frac{x(x^2+1)}{(x^2-1)^2}}$ (HINT: use logarithmic differentiation)

Problem 4. Compute the following limits.

(a) $\lim_{x\to 0} \frac{\sin x}{\sec x}$

(b) $\lim_{t\to 1} \frac{\cos(\frac{\pi}{2}t)}{\ln t}$.

(c) $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^{2x}$

(d) $\lim_{x\to 1} \left(\frac{1}{x-1}\int_1^x \sqrt{\tan^{-1} t}\, dt\right)$

(e) $\lim_{x\to 0^-} \frac{e^{1/x}}{x}$ (HINT: A change of variables may be helpful.)

Problem 5. Prove that the cubic polynomial $x^3 - 3x + c$ cannot have two different roots in the interval $(0,1)$. Explain your arguments precisely and clearly, citing all the theorems that you need.

Problem 6. Find the values of a and b for which the function

$$f(x) = a\log_e x + bx^2 + x$$

has extrema at points $x_1 = 1$ and $x_2 = 2$. Show that for the found values of a and b, the given function has a minimum at the point x_1 and a maximum at the point x_2.

Problem 7. Compute the following.

(a) $\int \sec x \tan x\, dx$

(b) $\int_1^2 \left(x^2 - \frac{1}{x^2}\right) dx$

Nishanth Gudapati, gudapati@math.jhu.edu

(c) $\displaystyle\int_1^3 \frac{e^{-1/t^2}}{t^3}\, dt$

(d) An equation for the tangent line to the curve $x^{1/2} + y^{1/2} = 3$ at the point $(4, 1)$.

Problem 8. Using an appropriately chosen substitution, compute the following integrals

(a) $\displaystyle\int \frac{x}{\sqrt{a^2 - x^4}}\, d\, x$

(b) $\displaystyle\int \frac{x(1 - x^2)}{1 + x^4}\, d\, x$

(c) $\displaystyle\int \frac{x + (\arccos 3x)^2}{\sqrt{1 - 9x^2}}\, d\, x$

(d) $\displaystyle\int \frac{1}{\sqrt{8 + 6x - 9x^2}}\, d\, x$

(e) $\displaystyle\int \frac{1}{x^2 + 2x + 3}\, d\, x$

Problem 9. Let R be the region in the first quadrant bounded by the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

and the x- and y-axes. Using any method you like, find the volumes obtained by rotating R (a) around the x-axis, and (b) around the y-axis. (HINT: It's up to you to figure out the limits of integration.)

Problem 10. Suppose that f is integrable on $[a, b]$, and let m and M be constants such that $m \leq f(x) \leq M$ for all $x \in [a, b]$. Using theorems from lecture or the text, prove that the average value of f on $[a, b]$ is between m and M. (HINT: A good way to get started is to write down how the average value of f is defined.)

Further, I recommend the following problems from the textbook

Section 6.5: 2, 8, 10, 18, 26

Section 8.1: 2, 4, 6 (skip the calculator part on 4 and 6), 8, 15

Section 8.2: 2(a), 4(a), 6, 8, 14.

Even though it is straightforward to extend the ideas we discussed in Section 6.2 to Section 6.3, since technically we couldn't discuss 6.3 in the class, it will not be part of the syllabus in the final exam. The full syllabus can be found here:
`http://www.math.jhu.edu/~gudapati/Calendar.pdf`

Please also review all the class notes and homeworks. It was a pleasure to be your instructor, I hope all of you do well in the exam.

Nishanth Gudapati, gudapati@math.jhu.edu