110.108 CALCULUS 1
FALL 2012
FINAL EXAM

Name: ____

Recitation section:

___ 1. Tues 1:30 (P. Shao)
___ 2. Tues 3:00 (P. Shao)
___ 3. Thurs 4:30 (B. Elder)
___ 4. Thurs 3:00 (Q. Guang)
___ 5. Thurs 1:30 (Q. Guang)

Work quickly and carefully, and write your solutions clearly. Please show your work; partial credit will be given generously.

Statement of ethics
I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ______________________________ Date: __________

Problem	Score
1	/15
2	/15
3	/10
4	/10
5	/10
6	/10
7	/12
8	/10
TOTAL	/92

Problem 1. [15 points] Compute the following limits.

(a) $\lim_{x \to \infty} \dfrac{4x^3 - 4x + 1}{3x^3 - 4x^2 + x + 6}$

(b) $\lim_{x \to 1} \dfrac{x^2 - 3x - 2}{x^2 + x - 1}$

(c) $\lim_{x \to \infty} (1 + 3x)^{1/\ln(x)}$

(d) $\lim_{x \to 2} \dfrac{\ln(x) - \ln(2)}{x - 2}$

(e) $\lim_{h \to 0} \dfrac{1}{h} \displaystyle\int_{x}^{x+h} \sin^3(t)\, dt$

Problem 2. [15 points] Compute the following.

(a) $\displaystyle\int_1^2 \frac{1}{x}\,dx$

(b) $\displaystyle\int -\frac{t^2}{(t^3+6)^3}\,dt$

(c) $\displaystyle\int_{-\pi/2}^{\pi/2} \cos\theta \cos(\pi \sin\theta)\, d\theta$

(d) $\displaystyle\int \frac{1-2v}{1+2v}\, dv$

(e) The average value of $f(x) = \dfrac{1}{\sqrt{5 - x^2}}$ on $[0, \sqrt{5}]$

Problem 3. [10 points] Find $\lim_{x \to 1} \dfrac{5x^2 - 15x + 10}{3x - 3}$ and give a δ-ϵ proof that you're right.

Problem 4. [10 points] A cylindrical canister is to be made with cardboard sides and metal top and bottom. If cardboard costs \$1 per square foot and metal costs \$2 per square foot, what are the height and radius of the cylinder of greatest volume that can be made with \$12?

Problem 5. [10 points] It's holiday time, and Morris, the Math Department Administrator, is whipping up a batch of his famous eggnog.

(a) Morris is pouring the eggnog into a hemispherical bowl of radius r. Show that when the eggnog in the bowl has depth h, it occupies a volume $V = \pi(rh^2 - \frac{1}{3}h^3)$. (HINT: Depending on how you set up the calculation, the formula $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ may be helpful.)

(b) If $r = 9$ inches and Morris pours the eggnog in the bowl at a constant rate of 2 cubic inches per second, what's the rate of change of h when $h = 3$ inches? (You may use the result of (a) even if you haven't actually done (a).)

Problem 6. [10 points] Let $I = [0, \pi]$, and let $f(x) = \displaystyle\int_1^{x+\cos x} e^{-t^2}\, dt$ for $x \in I$.

(a) Compute $f'(x)$.

(b) Find the critical points of f on I, and at each critical point, determine whether f has a local max, a local min, or neither. What is the absolute minimum value of f on I?

Problem 7. [12 points] Let R be the region in the plane bounded by the curves $y = 2 - x^2$ and $y = -2$. Set up, but *do not evaluate,* integrals to compute the following quantities.

(a) The area of R.

(b) The volume of the solid obtained by rotating R about the y-axis.

(c) The volume of the solid obtained by rotating R about the horizontal line $y = 3$.

(d) The arc length of the curve $y = 2 - x^2$ between its points of intersection with $y = -2$.

Problem 8. [10 points] In this problem we're going to prove the main theorem used in exponential growth and decay problems: If f is a differentiable function defined on an open interval I and there exists a constant k such that $f'(x) = kf(x)$ for all $x \in I$, then there exists a constant C such that $f(x) = Ce^{kx}$ for all $x \in I$.

(a) Suppose that f satisfies the hypotheses of the theorem, and define $g(x) = f(x)e^{-kx}$. Show that $g'(x) = 0$ for all $x \in I$.

(b) What does the result of (a) tell you about g? Use this to deduce the conclusion of the theorem. (To be clear, you may use the result of (a) even if you haven't actually done (a).)