Practice Midterm I

110.108 Calculus I for Engineers

Fall 2010

Directions: You have 50 minutes to complete this exam. No notes, books, or calculators are allowed. Show all work. Don't use any techniques that haven't been covered in class yet. This test might be too long as a few of the problems are computationally intensive but if you know your stuff, you should have little trouble. Herzlichen Glückwunsch!

1. Determine whether each of the following statements are TRUE or FALSE. If the statement is false, provide a counterexample showing why.

(a) The domain of all polynomials is all real numbers, $(-\infty, \infty)$.

(b) If $f(x)$ is a continuous function, then it is differentiable.

(c) If $f(x)$ and $g(x)$ are two functions, then $[f(x)g(x)]' = f'(x)g'(x)$.

(d) There exists a real number x such that $\sin\left(\frac{\pi x}{2}\right) = e^{-x}$.

(e) If the domain of f is D, then D is the range of f^{-1} where f^{-1} is the inverse of f.

2. (a) State the precise $\varepsilon - \delta$ definition of $\lim_{x\to a} f(x) = L$.

(b) Use the $\varepsilon - \delta$ definition from part (a) to prove that $\lim_{x\to 3} 4x + 5 = 17$.

3. Evaluate each of the following limits (without the use of derivatives!):

(a) $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

(b) $\lim_{x\to\infty} x - \sqrt{x+a}\sqrt{x+b}$

4. (a) What does it mean for us to say that a function $f(x)$ is continuous at the point a, in mathematical notation?

(b) For which value of k is the following function continuous at the point $x = 2$?

$$f(x) = \begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

5. (a) Write the definition of the derivative of a function $f(x)$.

(b) Use your answer for part (a) to compute the derivative of the function $f(x) = \frac{x^2}{\sqrt{x+2}}$ at the point $x = a$.

(c) Now use the standard rules for computing derivatives to comfirm the answer you found in part (b).

6. Using the fact that $\frac{d}{dx}(\ln x) = \frac{1}{x}$ $(x > 0)$, compute the derivative of $f(x) = x^x$ for $x > 0$. [HINT: Recall that $x^x = e^{\ln(x^x)} = e^{x\ln(x)}$.

7. (a) Let $f(x)$ be the function defined as

$$f(x) = \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n.$$

That is, $f(x)$ is defined by taking the limit as n goes to infinity of that expression. Find $f'(x)$ by first assuming that n is fixed and differentiating with respect to x, and then taking the limit as n goes to infinity.

(b) Based on your answer to part (a), what well-known function does $f(x)$ represent? [HINT: There is a very obvious relationship between $f(x)$ and $f'(x)$ which you can make use of.]