Practice Midterm II
110.108 Calculus I for Engineers
Fall 2010

Directions: You have 50 minutes to complete this exam. There are 7 problems, some with multiple parts. No notes, books, or calculators are allowed. Show all work. Don't use any techniques that haven't been covered in class yet. Good luck!

1. Determine whether each of the following statements are TRUE or FALSE. If the statement is false, provide a counterexample or explanation showing why.

(a) At a critical point for a function, $f'(x) = 0$.

(b) If $f(x)$ is a continuous and differentiable function and $x = a$ is a critical point, $f(a)$ is a maximum or a minimum.

(c) If $f'(x) > 0$ in some region, then $f(x)$ is increasing in that region.

(d) If $f(3) = 2$ and $f(0) = 1$, then $f'(x) = \frac{1}{3}$ somewhere in $(0, 3)$.

(e) $\int 2x + 1 \, dx = x^2 + x$

2. Find the derivative $\frac{dy}{dx}$ of the curve $x^3y = \sin(e^y x)$.

3. The minute hand of a clock is 6 inches long. Starting from noon, how fast is the area of the sector swept out by the minute hand increasing in in^2/min at any instant? [HINT: The area of a sector with angle θ and radius r is $\frac{r^2\theta}{2}$.]

4. (a) Write the equation for a local linear approximation of $f(x)$ around the point a; that is, write the linearization equation for the point x around the base point a.

(b) Use this equation to approximate $\sin(179°)$. [HINT: Convert to radians.]

5. Which is bigger, e^π or π^e? In this problem you will find out!

(a) Let $f(x) = \frac{\ln x}{x}$, $x > 0$. Find the critical points of $f(x)$.

(b) Determine whether the above critical points are maxima, minima, or neither, and use the derivative tests to sketch the graph of $f(x)$ for $x > 0$.

(c) What can you say about $f(e)$ relative to $f(\pi)$? (Is it smaller, larger, the same, can't tell,...)

(d) Use part (c) and some algebra to determine which is larger, e^π or π^e.

6. Show that the function $x^3 - 3x + m$ can't have two zeroes in the interval $[0, 1]$ no matter what m is; it might have zero or one but never two. [HINT: Suppose a and b are two roots in $[0, 1]$, so $f(a) = 0$ and $f(b) = 0$. Use Rolle's Theorem and deduce a contradiction.]

7. (a) What is wrong with the following use of L'Hôpital's rule? What should the limit really be?

$$\lim_{x\to 1} \frac{x^3 + x - 2}{x^2 - 3x + 2} = \lim_{x\to 1} \frac{3x^2 + 1}{2x - 3} = \lim_{x\to 1} \frac{6x}{2} = 3.$$

(b) Evaluate the following limit:

$$\lim_{x\to 1} \frac{x\ln(x) - x + 1}{x^3 + x^2 - 5x + 3}.$$