Mathematic 108, Fall 2015: Assignment #3

Due: **In your assigned section, either Tues., Sep. 22nd or Thurs., Sep. 24th.**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Using limit laws, show that the following functions are continuous at the given value a.

a) $f(x) = \frac{x^3-1}{x+1}$, $a = 1$.
b) $f(x) = \frac{\sqrt{x^2+4}+2}{x+2}$, $a = 0$.

Problem #2. Use continuity to evaluate the following limits.

a) $\lim_{x\to 0} \tan(x^2 - x)$.
b) $\lim_{x\to 1} \ln\left(\frac{3-x}{x^2+1}\right)$

Problem #3. Explain why the function is discontinuous at the given a and determine, if possible, the type of the discontinuity.

a) $f(x) = \frac{2x-1}{(4x+2)^2}$, $a = -\frac{1}{2}$.
b) $f(x) = \begin{cases} \frac{x+1}{1-\sqrt{-x}} & x < 0, x \neq -1 \\ 0 & x = -1, \end{cases}$ $a = -1$.
c) $f(x) = \begin{cases} \cos(x) & x < 0 \\ -\cos(x) & x \geq 0, \end{cases}$ $a = 0$.

Problem #4. Let f be continuous on $[2, 5]$. If f is zero only at $x = 5$ and $f(3) = -3$, then can $f(4) = 2$?

Problem #5. Determine value(s) c so that $f(x) = \begin{cases} -\frac{9}{x^2+c} & -3 \leq x \leq 2 \\ 2x - c & 2 < x < 5. \end{cases}$ is continuous on its domain.

Problem #6. Show that the function $f(x) = \begin{cases} x \sin\left(\frac{1}{x^2}\right) & x \neq 0 \\ 0 & x = 0. \end{cases}$ is continuous on $(-\infty, \infty)$.

Problem #7. Find the limit or explain why it doesn't exist.

a) $\lim_{x\to-\infty} \frac{3x^3+x^2-x+1}{x^3-1}$
b) $\lim_{x\to 0^-} \tan^{-1}\left(\frac{1}{x}\right)$

Problem #8. Using limit laws and definition of the derivative find $f'(a)$

a) $f(x) = -2x^2 + 3x - 1$, $a = 2$.
b) $f(x) = \sqrt{5 - x}$, $a = 1$.
c) $f(x) = x^{-1}$, $a = -1$.

Problem #9. Calculate $f'(0)$ for $f(x) = \begin{cases} 3 + x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 3 & x = 0. \end{cases}$

Problem #10. Let $g(x) = x^{2/3}$.

a) Show that $g'(0)$ doesn't exist.
b) Calculate $g'(a)$ for $a \neq 0$.
c) (Optional) Use either the book's definition on pg. 158 to show that g has a vertical tangent at $(0, 0)$ or use the definition from class to show that g has a vertical kink at $(0, 0)$. Indicate which you are using.

Book Problems.

a) Section 2.5: #4, #24, # 56
b) Section 2.6: #42, #58
c) Section 2.7: #8 , # 38
d) Section 2.8: #22, #26, #42