Mathematic 108, Fall 2015: Assignment #4

Due: **In your assigned section, either Tues., Sep. 29nd or Thurs., Oct. 1st**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Find the equation of the tangent line and of the normal line to the curve at the given point.

a) $y = 2x - x^2 + e^x$ at $(0, 1)$.
b) $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ at $(1, 0)$.

Problem #2. Find constants a, b so that $f(x) = x^2 + ax + b$ is tangent to the line $y = 2x - 3$ at $(2, 1)$.

Problem #3. Let $g(x) = xe^x$. Compute all $g^{(n)}(x)$ where n is a positive integer.

Problem #4. Suppose $f(x)$ is a differentiable function. Determine an expression for the derivative of the following functions

a) $g(x) = \frac{f(x)}{x}$.
b) $g(x) = \frac{1+f(x)}{1-f(x)}$.
c) $g(x) = x(f(x))^2$.

Problem #5. Suppose that g is differentiable.

a) Use the Quotient Rule to show $\frac{d}{dx}\left(\frac{1}{g(x)}\right) = -\frac{g'(x)}{g(x)^2}$.
b) Use the Chain Rule and the fact that $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$ to show the same thing.

Problem #6. Determine the constants m and b so that the function $f(x) = \begin{cases} xe^{1-x^2} & x \leq 1 \\ mx + b & x > 1 \end{cases}$ is differentiable on $(-\infty, \infty)$.

Problem #7. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x^2}\right) & x \neq 0 \\ 0 & x = 0. \end{cases}$

a) Compute $f'(x)$ for $x \neq 0$ and use this to determine $\lim_{x\to 0} f'(x)$.
b) Show that $f'(0) = 0$. How do you reconcile this with your answer in part a).

Problem #8. Compute the following limits

a) $\lim_{x\to 0} \frac{\sin(x)}{\sin(\pi x)}$.
b) $\lim_{x\to 0} \frac{\sin(x^2)}{x}$.
c) $\lim_{x\to 0} \frac{\cos(x)-1}{\sin(x)}$.

Problem #9. Let $h(x) = \sqrt{25 - 3f(x)}$ were $f(1) = 3$ and $f'(1) = 4$ determine $h'(1)$.

Problem #10. Determine the value(s) of α and β so that $f(x) = e^{\alpha x}\cos(\beta x)$ satisfies $f''(x) - 2f'(x) + 2f(x) = 0$ for all x. That is, so f is a solution to the differential equation $y'' - 2y' + 2y = 0$.

Book Problems.

a) Section 3.1: # 56, # 72
b) Section 3.2: #18, #46, #50
c) Section 3.3: # 12, # 18
d) Section 3.4: # 2, #36, # 64