201 LINEAR ALGEBRA, MIDTERM 2
April 11, 2011

NAME:

Section no:

TA:

. **There are 6 pages in the exam including this page.**

. **Write all your answers clearly. You have to show work to get points for your answers.**

. **Use of Calculators is not allowed during the exam.**

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Signature:

Date:

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1 (10)	
2 (10)	
3 (10)	
4 (10)	
5 (10)	
Total (50)	

1. *10 points*

(a) $T : P_2 \to P_2$ be the linear transformation defined by $T(f) = f + f''$. Let $\mathcal{S} = (1, x, x^2)$ be the standard basis for P_2. Find the $\mathcal{S}$-matrix A for T

(b) Let $\mathcal{B} = (1 + x, x + x^2, 1 + x^2)$ be another basis for P_2. Let B be the $\mathcal{B}$-matrix for the linear transformation T. Find the invertible matrix S such that $B = S^{-1}AS$.

2. *10 points* True or False. Justify your answer.

(a) There exists an invertible 2×2 matrix S such that $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = S^{-1} \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} S$.

(b) If $\vec{v_1}, \vec{v_2}$ is a basis for $\mathbb{R}^2$ then $T(\vec{v_1}), T(\vec{v_2})$ is a basis for $\mathbb{R}^2$ for any linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$.

3. *10 points* Find an orthonormal basis for $\text{Ker}(\text{Proj}_V)$ where $\text{Proj}_V : \mathbb{R}^4 \to \mathbb{R}^4$ is the orthogonal projection onto the subspace $V = \text{Span}\{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \end{pmatrix}\}$.

4. *10 points* True or False. Justify your answer.

(a) If A and S are orthogonal matrices, then $S^{-1}AS$ is orthogonal as well.

(b) Let A and B be two 2×2 matrices. If BA is orthogonal then A and B are orthogonal.

5. Find the least squares solution to the system $A\begin{pmatrix} x \\ y \end{pmatrix} = \vec{b}$ where $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

Find the orthogonal projection of $\vec{b}$ onto the subspace ImA.