Mathematics 201, Spring 2017: Assignment #1

Due: **In lecture, Friday, Feb. 10**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Write the augmented matrices that correspond to the following systems:

a) $\begin{cases} 2x + 3z = -3 \\ y - z = 1 \\ x + y + z = 0 \end{cases}$

b) $\begin{cases} 2x + 3z = -3 \\ y - z = 9 \end{cases}$

Problem #2. For the following augmented matrices, write the corresponding linear systems:

a) $\left(\begin{array}{ccc|c} 0 & 1 & 2 & -1 \\ 2 & 0 & 0 & 0 \\ 1 & -1 & 9 & 1 \end{array}\right)$

b) $\left(\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 2 & 0 & 0 & 7 & 1 \end{array}\right)$

Problem #3. Determine which of the following matrices are in reduced row echelon form. For those that fail to be in reduced row echelon form, explain why.

a) $\begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

Problem #4. Compute the reduced row echelon form of the following matrices:

a) $\begin{pmatrix} 2 & 6 & -2 \\ 3 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}$

b) $\begin{pmatrix} 3 & -3 \\ 1 & 1 \\ 0 & 0 \\ 2 & 4 \end{pmatrix}$

Problem #5. Use Gauss-Jordan elimination to find all solutions to the following linear systems:

a) $\begin{cases} 2x - 8y = 2 \\ 2x + y = 1 \end{cases}$

b) $\begin{cases} x - 2y - z = 2 \\ x - 3z = 2 \end{cases}$

c) $\begin{pmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 0 & -1 & 3 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -8 \\ 6 \\ 0 \end{pmatrix}$

Problem #6. Determine how the number of solutions of the following linear systems depend on the parameter k.

a) $\begin{cases} x - 2y = k \\ 3x + y = 1 \end{cases}$

b) $\begin{cases} x + y + 2z = 2 \\ x - z = k \\ 4x + y - z = 0 \end{cases}$

Problem #7. Compute the rank of the following matrices:

a) $\begin{pmatrix} 2 & 0 & 4 \\ -1 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 2 \\ -1 & 1 \\ 1 & 0 \end{pmatrix}$

Problem #8. Determine whether the following linear systems have no solution, a unique solution an infinite number of solutions, or if there is not enough information to decide:

a) A linear system whose coefficient matrix is 3×4 and of rank 2.

b) A linear system whose augmented matrix is 4×4 and of rank 4.

c) A linear system whose augmented matrix is 2×3 and of rank 2.

d) A linear system whose coefficient matrix is 3×3 and of rank 3.

Problem #9. Express the vector $\vec{b}$ as a linear combination of $\vec{v}_1$ and $\vec{v}_2$ or explain why this is impossible:

a) $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \vec{v}_1 = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 0 \\ -3 \\ -6 \end{pmatrix}$ b) $\vec{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \vec{v}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

Problem #10. Consider the linear system $A\vec{x} = \vec{b}$ where A has columns $A = \begin{pmatrix} \vec{v}_1 & | & \vec{v}_2 & | & \vec{v}_3 \end{pmatrix}$. Suppose your only information about this system is that $\vec{x} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ is a solution.

a) Give a solution to the system $A'\vec{x} = \vec{b}$ where $A' = \begin{pmatrix} \vec{v}_1 & | & \vec{v}_3 & | & \vec{v}_2 \end{pmatrix}$.

b) Give a solution to the system $A''\vec{x} = \vec{b}$ where $A'' = \begin{pmatrix} 2\vec{v}_1 & | & \vec{v}_2 & | & \vec{v}_3 \end{pmatrix}$.

Book Problems.

a) Section 1.1: #12, #22, #32
b) Section 1.2: #20, #22, #38
c) Section 1.3: #14, #26, #30, #36