Mathematics 201, Spring 2017: Assignment #2

Due: **In lecture, Friday, Feb. 17**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Determine *every* 3×3 matrix in reduced row echelon form which is of rank 2.

Problem #2. Suppose that A is a 2×2 matrix satisfying $A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = A \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

a) Determine *some* non-zero $\vec{x} \in \mathbb{R}^2$ so that $A\vec{x} = \vec{0}$. (Here $\vec{0}$ is the zero vector).

b) Determine *every* $\vec{x} \in \mathbb{R}^2$ so that $A\vec{x} = \vec{0}$.

Problem #3. Consider the following maps $T : \mathbb{R}^2 \to \mathbb{R}^3$. Determine whether they are linear and if so determine $[T]$, the matrix representation of T.

a) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 \\ x_2 \\ x_1 + x_2 \end{bmatrix}$.

b) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ x_1 + x_2 + 1 \\ x_1 \end{bmatrix}$.

Problem #4. Consider linear transformations $T : \mathbb{R}^3 \to \mathbb{R}^2$. Decide whether the following statements must be true for all T, for some T or for no T.

a) $T(\vec{x}) \neq \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for all $\vec{x} \in \mathbb{R}^3$.

b) $T(\vec{x}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ for some non-zero $\vec{x} \in \mathbb{R}^3$.

c) $T(\vec{x}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ only when $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$.

d) $T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Problem #5. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear map and that

$$T\left(\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

a) Determine, if possible, $T\left(\begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix}\right)$

b) Determine, if possible, $T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right)$

Problem #6. Find, if possible, all linear transformations, $T : \mathbb{R}^2 \to \mathbb{R}^2$ so that:

a) $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

b) $T\left(\begin{bmatrix} 4 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Problem #7. Let $R_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by rotating counterclockwise by θ radians and let $P_L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by orthogonally projecting onto the line L through the origin.

a) Using matrices, check that $P_L = R_\theta \circ P_1 \circ R_{-\theta}$ where here L makes angle θ with the x-axis and $P_1 : \mathbb{R}^2 \to \mathbb{R}^2$ is orthogonal projection onto the x-axis.

b) Give a geometric interpretation of why $P_L = R_\theta \circ P_1 \circ R_{-\theta}$.

Problem #8. Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ determine all 2×2 matrices, B, so that $BA = AB$.

Problem #9. Let A be a $n \times n$ matrix. Define $A^2 = AA$ be the product of A with itself.

a) Verify $A = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}$ satisfies $A^2 - 2A = I_2$.

b) Let $C = B^{-1}AB$ where B is some invertible 2×2 matrix and A is from a). Using properties of matrix multiplication, verify that $C^2 - 2C = I_2$.

Problem #10. Use Gauss-Jordan elimination to compute A^{-1} for the following matrices:

a) $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$.

b) $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & -1 \\ 2 & 1 & -3 \end{bmatrix}$.

Book Problems.

a) Section 2.1: #4, #6, #44
b) Section 2.2: #8, #10, #30
c) Section 2.3: #12, #24, #44
d) Section 2.4: #30