Mathematics 201, Spring 2017: Assignment #3

Due: **In lecture, Friday, Feb. 24**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Fix a $3 \times m$ matrix A and let $F_{1,2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

a) Check that $F_{1,2}$ has the property that the product $F_{1,2}A$ is obtained from A by swapping the first and second rows (Hint: Think about $m = 1$ case first).

b) Find $F_{1,3}$ and $F_{2,3}$, the 3×3 matrices so that $F_{1,3}A$ is obtained by swapping the first and third rows of A and that $F_{2,3}A$ is obtained by swapping the second and third rows of A.

c) Explain, in words, why $F_{1,2}^{-1} = F_{1,2}$.

Problem #2. Let A be a 2×3 matrix and suppose that $R = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ -1 & 1 \end{bmatrix}$ is a 3×2 matrix so that $AR = I_2$ (i.e., R is a right inverse of A). Find at least one solution to the following systems:

a) $A\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

b) $R\vec{x} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$.

Problem #3. Determine a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ so that:

a) $\ker(T) = \operatorname{span}(\vec{e}_1 - \vec{e}_2, \vec{e}_1 + \vec{e}_2 + \vec{e}_3)$.

b) $\operatorname{im}(T) = \left\{ t \begin{bmatrix} 2 \\ -1 \end{bmatrix} : t \in \mathbb{R} \right\}$.

Problem #4. Determine which of the following are linear subspaces of $\mathbb{R}^2$ or explain why they are not.

a) $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1^2 + x_2^2 + 1 = 0 \right\}$.

b) $W = \left\{ t \begin{bmatrix} 1 \\ 0 \end{bmatrix} : t \geq 0 \right\}$.

c) $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1 x_1 = 0 \right\}$.

d) $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1 - x_2 = 1 \right\}$.

Problem #5. Let L be an $n \times m$ matrix and R be an $m \times n$ matrix. Suppose $LR = I_n$ (i.e., L is a left inverse of R and R is a right inverse of L). Show the following:

a) $\ker(R) = \left\{ \vec{0} \right\}$.

b) $\operatorname{im}(L) = \mathbb{R}^n$.

Problem #6. Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. Determine whether $\vec{v}$ lies in $\operatorname{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$ and if it does express $\vec{v}$ as a linear combination of $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$.

a) $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

b) $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ -2 \end{bmatrix}$

Problem #7. Determine whether the following sets of vectors are linearly independent.

a) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

Problem #8. Consider 3×2 matrices A. Decide whether the following statements must be true for all A, for some A or for no A.

a) $\operatorname{im}(A) = \mathbb{R}^3$.

b) $\ker(A) = \left\{ \vec{0} \right\}$.

c) $\operatorname{im}(A)$ contains two linearly independent vectors $\vec{v}_1, \vec{v}_2 \in \mathbb{R}^3$.

d) $\ker(A) = \operatorname{span}(\vec{v}_1, \vec{v}_2)$ for some $\vec{v}_1, \vec{v}_2 \in \mathbb{R}^2$.

Problem #9. For the following matrices, A, find a basis of $\operatorname{im}(A)$.

a) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$.

b) $A = \begin{bmatrix} 2 & -4 & -1 \\ 1 & -2 & -2 \end{bmatrix}$.

Problem #10. For the following matrices A find a basis of $\ker(A)$.

a) $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & -1 \end{bmatrix}$.

b) $A = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & -1 & 2 \end{bmatrix}$.

Book Problems.

a) Section 2.4: #10, #28, #52
b) Section 3.1: #6, #8, #34
c) Section 3.2: #14, #32, #36, #46