Mathematics 201, Spring 2017: Assignment #4

Due: **In lecture, Friday, Mar. 3**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Suppose that A is a $n \times n$ matrix that satisfies the equation $A^3 + A^2 - 2A = -I_n$. Using this fact alone, deduce that A is invertible and determine a formula for A^{-1} in terms of I_n, A and A^2.

Problem #2. Fix a $n \times 3$ matrix, A, and let $F_{1,2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

a) Check that $AF_{1,2}$ is obtained from A by swapping the first and second columns of A.

b) Show that $\operatorname{im}(A) = \operatorname{im}(AF_{1,2})$.

c) Is it true that $\ker(A) = \ker(AF_{1,2})$?

Problem #3. Compute the rank and nullity of the following matrices:

a) $\begin{bmatrix} 2 & -2 & 0 & 4 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 3 & 1 \end{bmatrix}$.

b) $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$.

Problem #4. Let A be a 5×5 matrix. If $A = BC$ where B is a 5×3 matrix and C is a 3×5 matrix, can A be invertible?

Problem #5. Suppose that $\operatorname{rank}(A) \leq 2$. Can $\vec{v}, A\vec{v}, A^2\vec{v}, A^3\vec{v}$ be linearly independent?

Problem #6. Suppose A is a 4×4 matrix and that $A^2 = 0_{4\times 4}$ is the zero matrix.

a) Determine the relationship between $\ker(A)$ and $\operatorname{im}(A)$.

b) Use the rank-nullity theorem to show that $\operatorname{rank}(A) \leq 2$.

Problem #7. Consider a linear transform $T : \mathbb{R}^4 \to \mathbb{R}^2$. Determine whether the following is true for all T, for some T or for no T.

a) $\operatorname{rank}([T]) = \operatorname{null}([T])$.

b) $\operatorname{rank}([T]) = 2$ and $T(\vec{e}_1) = \vec{e}_2$.

c) $\operatorname{null}([T]) = 3$, $T(\vec{e}_1) = \vec{e}_1$ and $T(\vec{e}_2) = \vec{e}_1 + \vec{e}_2$.

d) $\operatorname{null}([T]) = 1$.

Problem #8. For the given matrix A and given (ordered) basis $\mathcal{B} = (\vec{v}_1, \ldots, \vec{v}_m)$ of $\mathbb{R}^m$ determine $[T]_{\mathcal{B}}$, the matrix of the linear transform $T(\vec{x}) = A\vec{x}$ with respect to $\mathcal{B}$.

a) $A = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}; \vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}; \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$.

Problem #9. Find an (ordered) basis $\mathcal{B}$ of $\mathbb{R}^2$ so that the matrix of

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 \\ x_2 \end{bmatrix} \text{ with respect to } \mathcal{B} \text{ is } \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}.$$

Problem #10. Suppose that a 3×3 matrix, A, satisfies $\ker(A) = \operatorname{span}\left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right)$. Determine an invertible 3×3 matrix, S, so that $S^{-1}AS$ has third column $\vec{0}$.

Book Problems.

a) Section 3.2: #34, #38
b) Section 3.3: #14, #22, #28, #30
c) Section 3.4: #12, #14, #22, #58