Mathematics 201, Spring 2017: Assignment #7

Due: **In lecture, Friday, Apr. 7th**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Consider the orthonormal vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4, \vec{u}_5 \in \mathbb{R}^9$. Compute

$$L = ||4\vec{u}_1 - 3\vec{u}_2 + \vec{u}_3 - 3\vec{u}_4 - \vec{u}_5||.$$

Problem #2. Apply the Gram-Schmidt algorithm to the following sets of vectors:

a) $\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -6 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

Problem #3. Compute the QR factorization of the following matrices:

a) $\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 4 & 25 & 0 \\ 0 & 0 & -2 \\ 3 & -25 & 0 \end{bmatrix}$

Problem #4. Find an orthonormal basis of the subspace $V \subset \mathbb{R}^3$ defined by $x_1 - x_2 + x_3 = 0$.

Problem #5. Find an orthonormal basis of $\mathrm{im}(A)$ when

$$A = \begin{bmatrix} 4 & 1 & 3 \\ -4 & 2 & -6 \\ 2 & 2 & 0 \end{bmatrix}.$$

Problem #6. Show that every 2×2 orthogonal matrix is of the form

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \text{ or } \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}.$$

Problem #7. Suppose that $A, B \in \mathbb{R}^{n\times n}$ are symmetric. Determine whether the following are symmetric, skew-symmetric or if there is not enough information to tell.

a) $3A + B$

b) $AB - BA$

c) $I_n + 2A + A^2$

d) AB^2.

Problem #8. Show that if A is similar to B, then $A^\top$ is similar to $B^\top$.

Problem #9. Determine (if possible) an orthogonal transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ so that

a) $T\left(\begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$

b) $T\left(\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$ and $T\left(\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$.

Problem #10. Let $\vec{v} = \begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix}$ and let $A = I_3 + \vec{v}\vec{v}^\top = \begin{bmatrix} 10 & 0 & -12 \\ 0 & 1 & 0 \\ -12 & 0 & 17 \end{bmatrix}$.

a) Show that A is similar to a diagonal matrix. (Hint: Consider $A\vec{v}$ and $A\vec{w}$ for $\vec{w}$ perpendicular to $\vec{v}$).

b) Show that there is an orthogonal matrix U so that $AU = UD$ for D a diagonal matrix.

Book Problems.

a) Section 5.1: #22, #26, #42
b) Section 5.2: #10, #28, #34, #36
c) Section 5.3: #4, #32, #40