LINEAR ALGEBRA

SECOND MIDTERM EXAM

DON'T PANIC!

JOHNS HOPKINS UNIVERSITY
SPRING 2013

You have **50 MINUTES**.
No calculators, books or notes allowed.

Academic Honesty Certificate. I agree to complete this exam without unauthorized assistance from any person, materials or device.

Signature: ______________________________ Date: ______________

Name: ______________________________ Section №: __________
(or TA's name)

Question	*Score*
1	
2	
3	
4	
5 (bonus)	

(1) Consider the linear transformation $\mathbf{R}^5 \to \mathbf{R}^4$ with matrix $A = \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 \end{bmatrix}$.

(a) [10 points] Determine a basis for the image of A.

(b) [10 points] Determine a basis for the kernel of A.

(c) [5 points] For which real numbers r (if any) does the equation $A\vec{x} = \begin{bmatrix} r \\ 0 \\ 0 \\ 1 \end{bmatrix}$ have a solution $\vec{x}$?

(2) (a) [12 points] Indicate which properties the given vectors have by writing *Yes* or *No* in each square. (Show your work in the margins or below the table.)

	linearly independent	(pairwise) orthogonal	orthonormal
$\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}1\\2\\-5\end{bmatrix}$ in $\mathbf{R}^3$			
$\begin{bmatrix}3/5\\4/5\end{bmatrix}, \begin{bmatrix}4/5\\-3/5\end{bmatrix}$ in $\mathbf{R}^2$			
$\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\-1\\-1\end{bmatrix}, \begin{bmatrix}1\\-1\\-1\\-1\end{bmatrix}$ in $\mathbf{R}^4$			
$(x-1)$, $(x-1)^2$, $(x-1)(x+1)$ in P_2			

(b) [13 points] Compute the inverse of the following matrix. Justify your answer.

[Hint: Intellect & romance triumph over brute force & cynicism.]

$$C = \frac{1}{2}\begin{bmatrix}1 & 1 & 1 & -1\\ 1 & 1 & -1 & 1\\ 1 & -1 & 1 & 1\\ 1 & -1 & -1 & -1\end{bmatrix}$$

(3) Consider the linear transformation $\mathrm{T}: \mathbf{R}^{2\times 2} \to \mathbf{R}^{2\times 2}$ given by the formula:

$$\mathrm{T}(M) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} M - M \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

(a) [20 points] Find the matrix B for T with respect to the basis:

$$\mathfrak{B} = \left(\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right)$$

(b) [5 points] Determine the dimension of the space of 2×2 matrices M such that $\mathrm{T}(M) = 0$.

(4) (a) [12 points] Orthonormalize—that is apply the Gram-Schmidt process to—these vectors in $\mathbf{R}^4$:

$$\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -3 \end{bmatrix}$$

(b) [13 points] Compute the 4×4 matrix (with respect to the standard basis) for the orthogonal projection onto the plane spanned by the vectors $\vec{v}_1$ and $\vec{v}_2$ above.

(5) [1 bonus point] This letter appears often in the textbook. Which letter of the alphabet is it?

[Hint: It's not the number **21***—half the number* **42***.]*