Mathematic 306, Final Exam Checklist

This is a rough list of topics to review. Please simply view this list as a good jumping-off point in your review: don't forget to carefully review your notes, homeworks, and the textbook.

Basic properties of ODEs

- Ordinary differential equations and initial value problems.
- What it means to solve a differential equation and an initial value problem.
- Simple models (falling object).
- Qualitative behavior of solutions; Direction/slope field.
- Order of a differential equation; Linear vs. non-linear ODE.
- Method of integrating factors for linear ODEs.
- Separable ODEs and separation of variables method.
- Autonomous first order ODEs.
- Equilibrium solutions; Stable vs. unstable equilibria of an autonomous ODE.
- Sketching solutions to autonomous equations.
- Examples of autonomous equations: Logistic growth model.

First order linear systems of ODEs

- What is a 2×2 or an $n \times n$ system of first order linear ODEs?
- Homogeneous vs. non-homogeneous systems.
- Direction fields; phase portraits.
- Eigenvalues and eigenvectors. Generalized eigenvalues.
- General solutions for real distinct eigenvalues vs. complex eigenvalues vs. repeated eigenvalues.
- Nature of equilibria, including stability, in each of the above cases (saddle/source/sink; spiral source/sink or center).
- Principle of superposition.
- Defective vs. non-defective matrices; Algebraic vs. geometric multiplicity of eigenvalues.
- Converting an order n ODE to a $n \times n$ system.
- Characteristic polynomial of a order n constant coefficient ODE.
- Method of undetermined coefficients (for non-homogeneous, constant coefficient ODEs)
- Variation of parameters formula.
- Spring model.

The Laplace transform and ODEs, including discontinuous forcings

- Improper integrals; piecewise continuous functions.
- Definition of Laplace transform; Inverse Laplace transform.

- Linearity of Laplace transform.
- Laplace transforms of simple functions (no need to memorize but should be ready to compute from the definition).
- Properties of the Laplace transform (no need to memorize but should know how to use).
- Derivation of some simple properties of Laplace transform from properties of integration.
- Computing the inverse Laplace transform using partial fraction decomposition.
- Applying the Laplace transform to solve ODEs.
- The Heaviside step function and indicator functions and their Laplace transforms.
- The Dirac delta "function".
- Expressing piecewise functions in terms of step functions and indicator functions.
- Applying the Laplace transform to solving constant coefficient ODEs with discontinuous forcing terms.
- Applying the Final Value Theorem

Non-linear autonomous systems

- Understand statement and consequences of local existence and uniqueness results; e.g. that distinct trajectories cannot cross.
- Determining the variational equation.
- Equilibrium solution of an autonomous system; Stable vs. Unstable vs. Asymptotically Stable equilibria.
- Determining (when possible) the qualitative nature (type, stability, etc.) of a critical point via properties of the linearized system.
- Plotting nullclines and using them to understand global behavior.
- Verifying that a quantity is conserved (or monotone, e.g. increasing/decreasing) along a trajectory for a non-linear system; i.e., verifying a function is a Lyapunov function.
- Applying conserved (or monotone) quantities to stability of equilibria and for determining equilibria; i.e., using Lyapunov functions.
- Gradient Systems.
- Hamiltonian Planar systems.
- Non-linear Pendulum
- Finding equations satisfied by trajectories (when possible) as solutions to Hamiltonian systems.
- Definition of α and ω-limit sets.