Math 306, Fall 2014: Assignment #10

Due: **Wednesday, December 3rd**

Instructions: Please ensure that your answers are legible and that sufficient steps are shown. Chapter numbers refer to the course text "Differential Equations, Dynamical Systems, and an Introduction to Chaos."

Problem #1. Chapter 9, # 6

Problem #2. Chapter 9, # 7 a) and c)

Problem #3. Chapter 9, # 8 a) and c)

Problem #4. Chapter 9, # 9

Problem #5. Chapter 9, # 15 (Hint: You should use the fact that any degree three polynomial has at least one real root)

Problem #6. Chapter 9, #16

Problem #7. Consider the planar systems

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} e^{x_2} \\ -2x_1 + ax_2^2 \end{pmatrix}$$

a) Show that this is a Hamiltonian system only for $a = 0$.
b) In this case determine the Hamiltonian.
c) Sketch phase portraits for $a < 0$, $a = 0$ and $a > 0$.
d) Determine the exact solution to the ODE with $a = 0$ and initial condition

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

(Hint: Use the initial conditions to find the level curve of the Hamiltonian, then use the Hamiltonian to turn the system into a one-dimensional ODE).

Problem #8. Consider the planar system

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} \cos x_1 \, (2x_1 - x_2) \\ x_1 + \frac{2x_2}{1+x_2^2} \end{pmatrix}$$

Restrict attention to the strip $|x_1| \leq \frac{\pi}{2}$

a) Sketch the phase portrait in the strip (Hint: Consider the nullclines).
b) For a point $\mathbf{X}_0 \neq 0$ in the strip what are $\alpha(\mathbf{X}_0)$ and $\omega(\mathbf{X}_0)$ the α and ω-limit sets.