

Midterm 1 Math 405 October 7, 2013
Show all work in a clear, concise and legible style.

1. (10 pts each) True or false; justify as much as you can.
 - a. The set of all sequences consisting of zeroes and ones is countable.
 - b. A sequence is convergent if and only if all of its subsequences are convergent.
 - c. $(n + 1)! \geq 2^n$ for all $n \in \mathbb{N}$.
 - d. The sup of an infinite bounded set S is the largest limit point of S .
 - e. The subset $(-1, 1) \setminus \{0\}$ of \mathbb{R} is open.
 - f. The countable union of closed intervals is closed.
2. (20 pts) Let $\{a_n\}$ be a Cauchy sequence. Show directly using the definition that the sequence $\{a_n^2\}$ is also a Cauchy sequence. Carefully justify all of the steps. You may use the result that a Cauchy sequence is bounded.
3. (20pts) Let $S = (-\infty, -1] \cup (1, 2) \cup \{3\}$. Find (5pts each)
 - a. The limit points of S .
 - b. $\partial S = \{\text{closure of } S\} \setminus \{\text{interior of } S\}$.
 - c. The isolated points of S .
 - d. The complement of S in \mathbb{R} ($S' = \mathbb{R} \setminus S$).