Midterm 2 Math 405 November 18, 2013

Show all work in a clear, concise and legible style.

Each problem is worth 25 points.

1. Let $f(x)$ be a bounded monotone increasing continuous function on $[a, b)$. Show that f extends to a continuous on [a,b] in the following steps:
a. Let $\{x_n\}$ be a sequence converging to b. Show that $L = \lim_{n\to\infty} f(x_n)$ exists.
b. Now suppose $\{y_n\}$ is another sequence converging to b with $M = \lim_{n\to\infty} f(y_n)$. Show that $M \leq L$. By symmetry $L \leq M$ and hence $L = M$.

2. Determine the constants k_1, k_2 so that the function

$$h(x) = \begin{cases} k_1 x - 5 & \text{if } x < 2 \\ 3 - k_2 x^2 & \text{if } x \geq 2 \end{cases}$$

is differentiable at $x = 2$. *Be sure to fully justify.*

3. Let f be a twice continuously differentiable (i.e C^2) function on $\mathbb{R}$.
a. State Taylor's theorem about the approximation of $f(x)$ near a point x_0 by a second order polynomial. Use Taylor's theorem to show that if $f'' < 0$, the graph of $f(x)$ lies on one side (below) its tangent line (the graph of its best linear approximation $l(x)$) in a small neighborhood of any x_0.

b. Still assuming that $f'' < 0$, show that the graph of $f(x)$ globally lies under its tangent line.

4. Let $f(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 4 & \text{if } x = 1 \\ 3 - x & \text{if } 1 < x \leq 2 \end{cases}$

State the Cauchy criterion for Riemann integrability and use it to show that f is Riemann integrable on [0,2]. You may use the theorem that a continuous function on a closed interval $[a, b]$ is Riemann integrable.