Practice Midterm 1 Math 405 Fall 2013

1. (10 pts each) True or false; justify as much as you can.

a. Two uncountable sets have the same cardinality

b. The numbers of the form $\frac{k}{3^n}$, $k, n \in \mathbb{N}$ are dense in $\mathbb{R}^+$.

c. For any $a \geq 0$, $(1+a)^n \geq 1 + na$ for all $n \in \mathbb{N}$.

d. Any sequence of decreasing nested intervals $I_1 \supseteq I_2 \supseteq \cdots \supseteq I_n \cdots$, with length of I_n decreasing to zero, has nonempty intersection, i.e. $\cap_{n=1}^{\infty} I_n \neq \emptyset$.

2. (20 pts) Let $\{s_n\}_1^\infty$ be a sequence of real numbers such that $s_n > 0$ for $n > k$. Suppose $s = \lim_{n\to\infty} s_n$ exists and is a finite real number. Prove that $s \geq 0$. Carefully justify all of the steps.

3. (20 pts) Let $\{a_n\}$ be a sequence of rationals and let $\{b_n\}$ be an equivalent sequence of rationals. If the sequence $\{a_n\}$ is Cauchy, show the sequence $\{b_n\}$ is also Cauchy.

4a. (10pts)Let $S = \{x \in \mathbb{R} : x^2 < x\}$. Find $\sup S$

4b. (10 pts) Let $a_n = (-1)^n(1 + \frac{1}{n})$, $n = 1, 2, \ldots$.

Find $\sup\{a_n\}$, $\inf\{a_n\}$, $\lim\sup\{a_n\}$, $\lim\inf\{a_n\}$.