Mathematic 405, Fall 2014: Assignment #1

Due: **Friday, February 7th**

Instructions: Please ensure that your answers are legible. Also make sure that sufficient steps are shown. Page numbers refer to the course text.

Problem #1. p. 13 # 1

Problem #2. p. 13 # 2

Problem #3. p. 13 # 4

Problem #4. p. 37 # 2

Problem #5. p. 37 # 3

Problem #6. p. 37 # 4

Problem #7. p. 37 # 5

Problem #8. Show that there is no $x \in \mathbb{Q}$ so that $x^2 = 6$.

Problem #9. The so-called Babylonian method for finding the square root of $S \in \mathbb{Q}^+$ is a recursive algorithm defined as follows: Start with any $x_0 \in \mathbb{Q}^+$ and define

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{S}{x_n}\right).$$

a) Show $S \leq x_{n+1}^2$ and hence $x_{n+1} \leq x_n$ for $n \geq 1$. Conclude that the closed intervals $I_n = \left[\frac{S}{x_n}, x_n\right]$ contain $\sqrt{S}$ in the sense that $\left(\frac{S}{x_n}\right)^2 \leq S$ and $S \leq x_n^2$ and form a decreasing nested sequence $I_{n+1} \subset I_n$.

b) Show that the lengths of I_n satisfy $|I_{n+1}| \leq \frac{1}{2}|I_n|$.

c) Show that $\{x_n\}$ is a Cauchy sequence over the rationals.

d) (optional) What goes wrong when $S < 0$?

(Note that one can take $S > 0$ real and obtain a a Cauchy sequence over the reals.)