Mathematic 405, Fall 2014: Assignment #3

Due: **Friday, February 21th**

Instructions: Please ensure that your answers are legible. Also make sure that sufficient steps are shown. Page numbers refer to the course text.

Problem #1. p. 84 # 1

Problem #2. p. 84 # 2

Problem #3. p. 84 # 4

Problem #4. p. 84 # 5

Problem #5. p. 84 # 10

Problem #6. p. 98 # 1

Problem #7. Dirichlet's approximation theorem implies that for any irrational number α there are an infinite number of integers p and q so that

$$\left|\alpha - \frac{p}{q}\right| < \frac{1}{q^2}.$$

a) Show that the set of rational numbers p/q where p and q are as above is also infinite.
b) Using the above fact and the fact that $|\sin x| \leq |x|$, show that 0 is a limit point of the sequence $\{\sin n\}$. (Hint: $\sin(m\pi) = 0$ for $m \in \mathbb{Z}$).