Math 405, Spring 2014: Assignment #6

Due: **Friday, March 28th**

Instructions: Please ensure that your answers are legible. Also make sure that sufficient steps are shown. Page numbers refer to the course text.

Problem #1. Let $f : D \to \mathbb{R}$ be uniformly continuous. Show that for every $x_0 \in \bar{D}$, that $\lim_{x\to x_0} f(x)$ exists (Hint: Construct a Cauchy sequence). Recall, this is not the case if f is only continuous.

Problem #2. Let $f : D \to \mathbb{R}$ be differentiable at $x_0 \in D$ and $g(x) = f(x_0) + m(x - x_0)$ be an affine function.

a) Show that $f - g = O(|x - x_0|), x \to x_0$.
b) Show $f - g = o(|x - x_0|), x \to x_0$ if and only if $m = f'(x_0)$.
c) (Extra Credit) If f is *not* differentiable at x_0, is it true that $f - g \neq O(|x - x_0|), x \to x_0$.

Problem #3. pg. 152 # 1.

Problem #4. pg. 152 # 3.

Problem #5. pg. 152 # 6.

Problem #6. pg. 152 # 7.

Problem #7. pg. 152 # 9.

Problem #8. pg. 163 # 2.