Math 405, Spring 2014: Assignment # 7

Due: **Friday, April 7th**

Instructions: Please ensure that your answers are legible. Also make sure that sufficient steps are shown. Page numbers refer to the course text.

Problem #1. Let $I = (a, b)$ be an interval. Show that if $f : I \to \mathbb{R}$ is differentiable and $|f'| < M < \infty$, then f is Lipschitz with Lipschitz constant M. Give an example to show that this is not possible without the uniform bound (i.e. give a differentiable function which is not Lipschitz.) Hint: use the Mean Value Theorem.

Problem #2. Let $I = (a, b)$ be an interval. Suppose that $f : I \to R$ is differentable. Show that for any compact interval $I_\delta \subset I$ with $I_\delta = [x_0 - \delta, x_0 + \delta]$, one has

a) $f(I_\delta)$ is a compact interval.
b) $|f(I_\delta)| - 2|f'(x_0)|\delta = o(\delta), \delta \to 0$. Here $|I|$ means the length of a compact interval I, so $|I_\delta| = 2\delta$. That is, $|f'(x_0)|$ measures the extent to which f distorts length near x_0.
c) (Optional) Give a "geometric" interpretation of the chain rule using the above observation.

Problem #3. p. 163 # 4

Problem #4. p. 163 # 8

Problem #5. p. 163 # 10

Problem #6. p. 163 # 13

Problem #7. p. 176 # 1

Problem #8. p 176 # 4