

Math 645, Fall 2017: Assignment #1

Due: **Thursday, September 21th**

Problem #1. Let M and N be differentiable manifolds, show that there is a differentiable manifold structure on the cartesian product $M \times N$ so that the two natural projection maps $\pi_M : M \times N \rightarrow M$ and $\pi_N : M \times N \rightarrow N$ are smooth.

Problem #2. Let M and N be topological spaces and $\phi : M \rightarrow N$ be a homeomorphism. Show that if N is a differentiable manifold, then there is a smooth atlas on M so that ϕ is diffeomorphism.

Problem #3. Let \mathbb{R} denote the differentiable manifold coming from the standard atlas on \mathbb{R} . Let $\phi : \mathbb{R} \rightarrow \mathbb{R}$ be given by $\phi(x) = x^3$ and let \mathbb{R}' denote the differentiable manifold (i.e., the structure on \mathbb{R}) that makes ϕ a diffeomorphism. Describe $C^\infty(\mathbb{R}') \cap C^\infty(\mathbb{R})$.

Problem #4. Show that $\mathbb{R}\mathbb{P}^2$ is non-orientable.

Problem #5.

- a) Let M and N be differentiable manifolds. Prove that $M \times N$ is orientable if and only if both M and N are orientable.
- b) Prove that TM is orientable (even if M is not).
- c) Prove that $T\mathbb{R}\mathbb{P}^2$ is not diffeomorphic to $\mathbb{R}\mathbb{P}^2 \times \mathbb{R}^2$.