5x5 2x2 rank=1

$\begin{matrix} 1 & 2 \\ 0 & 0 \end{matrix}$ $\quad x+2y=0$
$\quad 0x+0y=0$ $\quad x=-2y$

$Ax=0$

$Ax=0$

0 0 0 0 0

A rank $A=4$
5x5

4x3 $\underbrace{\underset{4\times 3}{A}}\, x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$\underset{4\times 3}{A}$ $\quad Av=Aw$

...

$$A(v-w) = 0$$

rank 3 sol. is unique

$$v = w$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{array}{ccc} 1 & -3 & 0 \end{array}$$

$$x + 2y + 3z = 1$$

$$4x + 5y + 6z = 2$$

$$0x + 0y + 0z = 3$$

$$0 = 3$$

$$\begin{array}{cccc} -5 & 2 & 1 & 0 \end{array}$$

$$\begin{array}{ccc} 1 & -3 & 0 \end{array}$$

$$x + y - z = -2$$

$$3x - 5y + 13z = 18$$

$$x - 2y + 5z = K$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 3 & -5 & 13 & 18 \\ 1 & -2 & 5 & K \end{array}\right] \xrightarrow{-3(1)+(2) \quad (3)-(1)} \begin{array}{cccc} 1 & 1 & -1 & -2 \\ 0 & -8 & 16 & 24 \\ 0 & -3 & 6 & K+2 \end{array}$$

-8

$$\begin{array}{cccc} 1 & \cancel{1} & -1 & -2 \\ 0 & 1 & -2 & -3 \\ 0 & -3 & 6 & \underline{K+2} \end{array} \quad \xrightarrow{(1)-(2) \quad 3(2)+(3)} \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & \underline{K-7} \end{array}\right]$$

$K-7=0$ if you have a sol..

$\underline{K-7 \neq 0}$ no solution

$0 \neq K-7$

$K \neq 7$ inconsistent

$K=7$

$$\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -3 \end{array}$$

$$\begin{aligned} x \quad\quad + z &= 1 \\ y - 2z &= -3 \end{aligned}$$

$x = 1 - z$

$y = 2z - 3$

$z = z$

$$\begin{array}{cccc} 1 & 0 & 0 & a \\ 0 & 0 & 1 & b \end{array}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1-z \\ 2z-3 \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} + z\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$f'(t) = b + 2ct \quad f'(2)$

$f(t) = a + bt + ct^2$

$(1,1) \quad (3,3) \quad f'(2) = 3$

$t \quad f(t)$

$$a + b + c = 1$$
$$a + 3b + 9c = 3$$
$$b + 4c = 3$$

$(2) - (1)$

$$\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 3 \\ 0 & 1 & 4 & 3 \end{array} \qquad \begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 2 & 8 & 2 \\ 0 & 1 & 4 & 3 \end{array}$$

$(3) - (2)$

$$\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 1 & 4 & 1 \\ 0 & 1 & 4 & 3 \end{array} \qquad \begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{array}$$

$$0 = 2$$

$$\cancel{x + \quad - z = 2}$$

$$x + y - z = 2$$
$$x + 2y + z = 3$$
$$x + y + (k^2 - 5)z = k$$

$$\begin{array}{cccc} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & k^2-5 & k \end{array}$$

(2)−(1)

$$\begin{matrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k^2-4 & k-2 \end{matrix}$$

(3)−(1)

(1)−(2)

$$\begin{matrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k^2-4 & k-2 \end{matrix}$$

if $k^2-4 = (k+2)(k-2) = 0$

$k = \pm 2$

if $k = -2$

$$\begin{matrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -4 \end{matrix}$$

Inconsist.

$0 = -4$

$k = 2$

$$\begin{matrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$z = s$

$x \quad -3z = 1$ $\qquad x = 1 + 3z$

$y + 2z = 1$ $\qquad y = 1 - 2z$

$z = z$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+3s \\ 1-2s \\ s \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

What if $K^2-4 \neq 0$

$$\begin{matrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & K^2-4 & K-2 \end{matrix}$$

$\neq 0$

$$\frac{K-2}{K^2-4} = K+2$$

$$\begin{matrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \cancel{K+2}\ \frac{1}{K+2} \end{matrix}$$

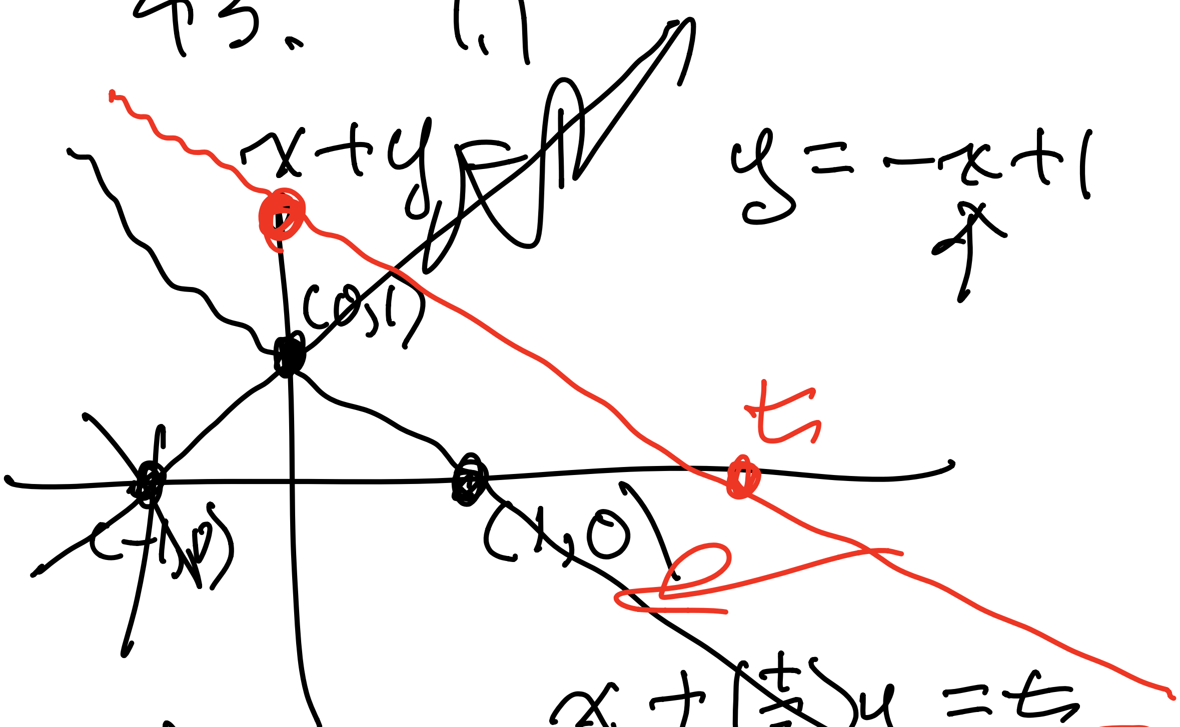
$$\begin{matrix} (1)+3(3) & 1 & 0 & 0 & 1+\frac{3}{K+2} \\ (2)-2(3) & 0 & 1 & 0 & 1-\frac{2}{K+2} \\ & 0 & 0 & 1 & \frac{1}{K+2} \end{matrix}$$

$$x = 1 + \frac{3}{K+2}$$

$$y = 1 - \frac{2}{k+2}$$

$$x = \frac{1}{k+2}$$

43. $(1,1)$



$$\left(\frac{t}{2}\right)y = -x + t$$

$$y = -\frac{2x}{t} + 2$$

$$0 = -\frac{2}{t}x + 2 \qquad \frac{2}{t}x = 2$$

$$-1 \qquad -\frac{2}{t} \qquad t \neq 2$$

$$x + y = 1$$
$$x + \frac{t}{2} y = t$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & \frac{t}{2} & t \end{array}\right]$$

(2) − (1)
$$\begin{matrix} 1 & 1 & 1 \\ 0 & \frac{t}{2}-1 & t-1 \end{matrix}$$

$$\frac{t}{2}-1 \neq 0$$

$t \neq 2$

$$\begin{matrix} 1 & 1 & 1 \\ 0 & t-2 & 2(t-1) \end{matrix}$$

(1) −

$$\begin{matrix} 1 & 1 & 1 \\ 0 & 1 & \frac{2(t-1)}{t-2} \end{matrix}$$

~~(2)~~ (1) − (2)

$$\begin{matrix} 1 & 0 & 1 - \frac{2(t-1)}{t-2} \end{matrix}$$

$$0 \quad 1 \quad \frac{2(t-1)}{t-2}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 - \frac{2(t-1)}{t-2} \\ \frac{2(t-1)}{(t-2)} \end{pmatrix}$$

$$\frac{t-2-2t+2}{t-2} = \frac{-t}{t-2}$$

$$= \begin{pmatrix} \frac{-t}{t-2} \\ \frac{2(t-1)}{t-2} \end{pmatrix} = \frac{1}{t-2} \begin{pmatrix} -t \\ 2(t-1) \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad Bx = 0$$

$$\text{rref} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$
\begin{aligned}
a \quad - c &= 0 \\
b + c &= 0
\end{aligned}
$$

$$
\begin{aligned}
x + y \quad &= 0 \\
x + 2y + z &= 0 \\
0 + y + z &= 0
\end{aligned}
$$

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad B\vec{X} = \vec{0}$$

$$A\vec{x} = \begin{bmatrix} v_1, & v_2 \cdots & v_m \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}$$

$$= x_1\vec{v}_1 + x_2\vec{v}_2 + \cdots + x_m\vec{v}_m$$

$$\vec{v}_k = \begin{pmatrix} v_{1,k} \\ \vdots \\ v_{n,k} \end{pmatrix} \quad \begin{pmatrix} V_{1,k} \\ \vdots \end{pmatrix}$$

$v_{2,k}$

$\vdots$

$v_{n,k}$

$$v_{11}x_1 + v_{12}x_2 + \cdots v_{1m}x_m =$$
$$v_{21}x_1 + v_{22}x_2 + \cdots + v_{2m}x_m$$
$$a_i \quad \vdots$$
$$v_{n1}x_1 + v_{n2}x_2 + \cdots \quad v_{nm}x_m$$

v_1

x

$\vdots$

$a_{ij} = v_{ij}$

$$x_1\vec{v}_1 + x_2\vec{v}_2 + \cdots x_m\vec{v}_m$$

$$AX = \begin{pmatrix} - w_1 - \\ - w_2 - \\ - w_n - \end{pmatrix}\begin{pmatrix} X \end{pmatrix}$$

$W_1 \circ X$

$a_1 \quad a_2 \quad a_3 \cdots a_n$ $\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

$a_1x_1 + a_2x_2 \cdots a_nx_n$

$$A = (a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & \cdots & & \\ a_{31} & & & \\ \vdots & & \cdots & \\ a_{n1} & & & \end{pmatrix}$$

$4 \times 3 \quad \text{rank} = 3$

$Av = Aw \Rightarrow v = w.$

$\Leftrightarrow$

$A(v-w) = 0$

$Av - Aw = 0$

$\Downarrow \quad AX = 0 \quad v = w.$

$AX = 0$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \\ a_{41} & 0 & 0 \end{pmatrix} \quad \mathrm{rref}(A) = 0$$

$$\mathrm{rref}(A) \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$x = 0$
$y = 0$
$z = 0$

$m \times n$

$m \geq n$ $\quad$ rank $= n$ $\quad \begin{pmatrix} 1 & & & & \cdots \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \\ & & 0 & & \end{pmatrix}$

$m < n$ $\quad AX = 0$

$\overline{Ax = y \neq 0}$

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

1 1 1

~~0 0 0~~ rank = 1

~~2×3~~ ~~0 1 7~~

$$x_2 + 2x_4 + 3x_5 = 0$$

$$4x_4 + 8x_5 = 0$$

$$\begin{matrix} 0 & 1 & 0 & 2 & 3 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{4} & \cancel{8} \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{matrix}$$

$$x_2 - x_5 = 0$$

$$x_4 + 2x_5 = 0$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_5 \\ x_3 \\ -2x_5 \\ x_5 \end{pmatrix}$$

$$= x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 0 \\ 1 \\ 0 \\ -2 \\ 1 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$