A Report to the Washington State Board of Education:

Second Review of Washington K–12 Mathematics Standards

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Strategic Teaching
Better learning by design
1573 Millersville Road
Millersville, MD 21108

The Washington State Board of Education (SBE) is committed to making sure Washington's mathematics standards compare favorably with the best in the world. The Board asked Strategic Teaching to review the Office of the Superintendent of Public Instruction's (OSPI) February 29, 2008 version of the K–12 Washington State Mathematics Standards to be sure this is this case.

What follows is a broad review, completed quickly. Pressure from the legislative calendar allowed only a week to do the work. While the importance of the topic merits considerably more time, we are satisfied we have answered the two important questions: "Do Washington's new standards rank among the best? If not, what needs to happen to make them so?"

We need to break our response to these two questions into grades K–8 and 9–12.

The new mathematics standards for grades K–8 are very close to excellent. These standards *do* compare favorably with the best in the nation and the world. The *Performance Expectations* (*Expectations)* are specific, measurable, important mathematical topics that are both focused at particular grades and developed across grade levels. The problems that remain regarding clarity, mixing instructional methods in the standards, and polishing the language can (and should) be easily fixed with essentially a strong edit.

However, the high school standards need more than this. While they are much improved from OSPI's January version, further revision is needed. Some areas, such as occasional imprecision of language, is similar to grades K–8 and just as easily fixed. Other areas, such as missing content and content organization, are more problematic. We expand below on both K–8 and high school.

We first want to commend the substantial work of Washington educators and community leaders, OSPI, and the Dana Center. Washington has broken new ground in its approach to organizing grade level content by priorities rather than mathematical strands. The writing teams were inclusive, the stakeholder feedback extensive. The document clearly is thoughtful and written with mathematical expertise.

Grades K–8

The K–8 standards need a substantive edit to tighten the language and move instances of pedagogy from the *Expectations* to the *Explanatory Comments and Examples* (*Comments*) side of the document. The important content is already included at grade levels that find the balance between rigor and reason.

We examine grades K–8 from the perspective of the topics posed by the original *Review and Recommendations*, the Washington Mathematics Panel's feedback to *Follow-up to the Review and Recommendations,* and our own analysis.

Some recommendations — such as "restructure the document" — were accomplished by the January 29, 2008, draft. Other recommendations — such as "create small writing teams" — are no longer pertinent. This second review will not revisit these and concentrate only on the remaining topics. Notice that the recommendations have been modified to reflect the Mathematics Panel's feedback and the expanded purpose of this review.

Recommendation 1: Set higher expectations for Washington's students by fortifying content and increasing rigor. *Additionally, set the level of rigor to compare favorably to other exemplar standards documents and to high-achieving countries included in the original Strategic Teaching study for the SBE.*

OSPI's standards seem to be written at about the same level of rigor as the standards of other states considered to be exemplary.

We would like to make one important point about rigor. It is not just about teaching topics at the earliest grade levels. It also is about teaching topics in depth and expecting that students use the content in demanding ways. Washington does these things. It focuses topics at grade levels and covers those topics thoroughly. It consistently requires students to solve multistep, contextualized problems with the content. This cannot be said of the exemplar standards from the other states.

Time does not allow an exhaustive mapping of OSPI's standards to the exemplar documents in the original study,[1] but a judicious spot check of key topics suggests they align to other states. International comparisons and comparisons to assessments were not made because of the complexities related to the variability of documents, document types and grade-level availability.

Here is one example of how the new standards map to other states. Washington introduces linear relationships in grade 5 and linear equations at grade 7. Massachusetts does these things by grades 6 and 8, respectively; California in grades 5 and 6, respectively; and Indiana in grades 5 and 6, respectively.

Another example is that Washington and Massachusetts expect students to find the perimeter of two-dimensional figures in grade 3 and both the area and perimeter of rectangles in grade 4; California and Indiana require students to find perimeters and to work with areas in grade 3. Although it is not an exact match, we find Washington comparable.

And last, Washington introduces fractions in grade 2, as do California, Indiana and Massachusetts. Although two of the states expect slightly more at grade 2 — locate fractions on a number line and recognize and name unit fractions —by grade 5 all of the states expect students to add and subtract with fractions, decimals and mixed numbers.

Only California goes beyond the other states and also expects students to multiply and divide fractions in grade 5, but California is alone in expecting all students to take Algebra I in grade 8. As a result, comparisons break down for grades 7 and 8.

Washington's grade 7 standards include the substantive topics of rational numbers, linear equations and proportionality, including slope. Washington's grade 8 standards include linear equations and inequalities as well as basic geometric theorems such as the Pythagorean theorem. Since virtually all students can opt to take Algebra I in grade 8 — this is typically when students begin to

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[1] (1) California State Standards, (2) Massachusetts State Standards, (3) Indiana State Standards, (4) Singapore Curriculum, (5) Finland Standards, (6) *Curriculum Focal Points,* (7) the National Assessment for Education Progress (NAEP), (8) the American Diploma Project, and (9) the Washington College Readiness Math Standards

accelerate — we do not find the difference in rigor troubling at these grade levels.

Recommendation 2: Make clear the importance of all aspects of mathematics: mathematics content, including the standard algorithms; the conceptual understanding of the content; and the application of mathematical processes within the content. *Additionally, differentiate the process strands so there is less repetition across grade levels, and better embed the processes into the content standards.*

OSPI's standards are a balance of conceptual understanding and mathematics content and make clear students must learn their math facts and standard algorithms. Guidance about technology use is provided and the *Core Processes* are differentiated, although we feel that the last two areas could be strengthened.

In our review of the January 31 version of the new standards, we called for clarity around the mastery of mathematics facts and standard algorithms. In the *Expectations* of the February 29 version, it is made clear that students are to memorize their addition and multiplication facts and be fluent with the standard algorithms for whole-number arithmetic:

2.2.B Quickly recall basic addition facts and related subtraction facts for sums through 20.

3.4.D Fluently add and subtract whole numbers accurately using the standard regrouping algorithms.

4.1.A Quickly recall multiplication and related division facts through 10 × 10.

4.1.F Fluently multiply up to a three-digit number by one- and two-digit numbers accurately using the standard multiplication algorithm.

5.1.C Fluently divide numbers of up to four digits by one- and two-digit divisors accurately using the standard long division algorithm.

These essential skills are now unambiguously presented in the standards with complete clarity.

We believe the document can be further strengthened with additional or replacement right-hand comments. To take one example, *Comments* should include a division problem solved using the traditional method. It also should explain that the long division algorithm is an excellent example of many topics — iteration, approximation, place value, inverse operations. Other ways to teach and build understanding of division, such as partial products, also can be included along with an explanation of how they help build understanding of place value.

As we had hoped from the beginning, the standards do this while making clear that students are to understand what addition, subtraction, multiplication and division mean. The standards also leave room for teachers to teach and students to learn using multiple methods, such as the modeling multiplication example for 4.1.C.

These are not the only core skills students need. In addition to whole-number arithmetic, students must be able to use the four arithmetic operations fluently with fractions and decimals. These standards are explicit:

5.2.E Fluently add and subtract fractions, including mixed numbers.

5.2.F Fluently add and subtract decimal numbers.

6.1.D Fluently multiply and divide non-negative fractions.

6.1.E Fluently multiply and divide non-negative decimals.

OSPI's standards have the basic arithmetic skills well under control for whole numbers, positive fractions and positive decimals by the end of grade 6.

In addition to the guidance provided in the *Expectations*, the introduction to the standards gives guidance about the use of technology. No one can argue with the statement, "TECHNOLOGY SHOULD BE USED when it can enhance the attainment of the mathematics learning goal, and TECHNOLOGY SHOULD NOT BE USED if it undermines the attainment of the mathematics learning goal."

It goes on to specify that "elementary students are expected to know facts and basic computational procedures — skills that should be implemented without calculators." After that, it says, "The choice of what tools to use and how to teach are appropriately left to the judgment of professional teachers."

Technology should not replace student calculation, but it also is important that technology be used as an instructional tool. Particularly at the high school level and in statistics, technology allows students access to problems and modeling that would not be possible otherwise. Students need to know how to use spreadsheets and graphing calculators, for example.

All this is to say there are times when technology is inappropriate and times when it should be required. We believe there are some instances when the *Expectations* should be more explicit about when this is so.

We have mentioned that the *Core Content, Additional Key Content* and *Core Processes* combine to produce a document in which both the content and the mathematical processes have emphasis. But thus far, we have not spoken to the quality of the *Core Processes.*

We found the individual *Expectations* for *Core Processes* to be well written.

We also examined the *Core Processes* to determine whether or not OSPI achieved its goal of differentiating by grade level. Distinctions are made in the explanatory text at the top of the page and in the *Comments* on the right of the page. These are descriptive, and they clarify what is expected at each grade level. However, the *Expectations* vary little from grade to grade.

Compare, for example, the first standard in kindergarten to the first standard in grade 8.

Kindergarten:

- Identify the question(s) asked in a problem and any other questions that need to be answered in order to solve the problem.

Grade 8:

- Analyze a problem situation to determine the questions(s) to be answered.

Describing what we want students to do in terms of reasoning, problem solving and communicating is probably the most difficult part of writing mathematics standards. Many states take the path of least resistance and write broad statements or introductory text explaining the importance of the processes and an admonishment that students should reason, solve problems and communicate in all of the mathematical strands.

We offer some example standards from Indiana to show what another state has done.

In kindergarten, students are asked to:

- Choose the approach, materials, and strategies to use in solving problems.
  - Example: Solve the problem: "There are four blocks on the table and a box of blocks that is closed. The teacher says that there are five blocks in the box. Find the number of blocks in all, without opening the box." Decide to draw a picture.

- Explain the reasoning used with concrete objects and pictures.
  - Example: In the first example, draw a picture of the four blocks that you can see, and then draw five more blocks for the ones that you cannot see.

In grade 4, students are asked to:

- Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, tools, and models to solve problems, justify arguments, and make conjectures.
  - Example: Solve the problem: "Find a relationship between the number of faces, edges, and vertices of a solid shape with flat surfaces." Try two or three shapes and look for patterns. Make a table to help you explain your results to another student.

- Decide when and how to break a problem into simpler parts.
  - Example: In the first example, find what happens to cubes and rectangular solids.

- Apply strategies and results from simpler problems to solve more complex problems.
  - Example: In the first example, use your method for cubes and rectangular solids to find what happens to other prisms and to pyramids.

Not every expectation changes at every grade level in Indiana, but they develop over time and an attempt is made to tie them to the content standards at that grade level.

All this said, given the timeframe for this work, OSPI current effort seems a good compromise. Particularly given that other exemplar documents do not offer this kind of differentiation for process standards, this is a revision OSPI can offer in the future that likely to be acceptable to teachers. It is wise to spend its time on other concerns.

Recommendation 3: Identify those topics that should be taught for extended periods at each grade level, and better show how topics develop over grade levels. *Additionally, be sure that the* Additional Key Topics *are the right ones and that they do not distract from the* Core Content.

The new standards are superior to the standards of other states considered exemplar in their organization of the content by grade level priority rather than mathematical strands. In general, the topics are allocated across grade levels and between *Core Content* and *Additional Key Topics (Key Topics*) appropriately, although there is a lot of content at grades 3 and 4.

There is a lot of concern about the *Additional Key Topics (Key Topics*) and how much time they will be assigned in the classroom. Some worry that topics in *Key Topics* will grow and displace *Core Content*. Others are concerned that important content in *Key Topics* will be lost and not get the attention it deserves.

Although we understand the concerns, we see the structure of the document as meeting the intent of the original recommendations and meeting it well. The *Core Topics* are the important topics that deserve extended attention in grades K–8. Our understanding is that *Key Topics* are equally important, testable expectations that deserve a place in the curriculum but not extended time. This should be made clear in the standards introduction.

For the most part, we see the total of the *Expectations* and the *Key Topics* as representing a teachable amount of content within a grade level. Grades 3 and 4, however, have numerous topics in this section, particularly in *Key Topics* that deserve another review.

Recommendation 4: Increase the clarity, specificity and measurability of the Grade Level Expectations*. Additionally, be sure that the standards are standards and not curriculum or pedagogy.*

While most of the *Expectations* are well written, some lack clarity and others mix pedagogy into the standards.

As stated in the opening, for grades K–8, the February 29 version of the standards is very strong. We see the *Expectations* as generally clear, specific and measurable. We offer some examples above related to basic skills, but there are many others:

- 2.1.E Describe a number up to 1,000 in terms of how many ones, how many tens, or how many hundreds it includes.
- 4.4.D Graph and identify points in the first quadrant of the coordinate plane using ordered pairs.
- 6.1.F Describe the effect of multiplying or dividing a number by one, by zero, by a number between zero and one, and by a number greater than one.
- 8.1.B Solve one- and two-step linear inequalities and graph the solutions on the number line.

However, not all of the *Expectations* are this well written.

Two issues which complicate standards writing are “understanding” and “pedagogy.” Crafting a standards document that captures the importance of student understand without crossing the line into instruction is not easy. Understanding is much easier to see in the classroom than it is to describe in standards. And the difference between language that describes understanding and language that describes instructional practice can be debated.

This speaks to Jere Confrey’s statement that “Standards inherently involve tensions. They are goal statements about which different people, even different experts, will have varied opinions. They require negotiations, and represent compromises among varied legitimate participants and groups."[2]

In our mind there are many cases in which the *Expectations* cross into pedagogy, others in which the language should be tightened and a small number that are muddled. We give several examples here, but it is not an exhaustive list.

We also offer suggestions for improvement.

Clarity

1.2.H Solve and create a story problem that matches an addition or subtraction expression or equation using physical objects, pictures, or words.
1.2.H (revised) Solve a story problem that matches an addition or subtraction equation and create a story that matches an addition or subtraction expression or equation.

2.2.H Solve equations in which the unknown number and the equal sign appear in a variety of positions.
2.2.H (revised) Solve equations in which the unknown number and the equal sign appear in a variety of positions relative to each other.

4.4.F Describe and compare the likelihood of events.
4.4.F (revised) Describe and compare the likelihood of two events.

Pedagogical Issues

3.2.A Represent fractions that have denominators ranging from 2 to 12 using physical objects, pictures, numbers, and words, and translate among representations.
3.2.A (revised part 1) Represent fractions that have denominators ranging from 2 to 12 as parts of a whole, parts of a set, or locations on a number line.
(revised part 2) Translate among representations of fractions that have denominators ranging from 2 to 12.

5.2.A Represent addition and subtraction of fractions and mixed

[2] Confrey, Jere, "Tracing the Evolution of Mathematics Content Standards in the United States: Looking Back and Projecting Forward towards National Standards," a paper prepared for the Conference on K–12 Mathematics Curriculum Standards, sponsored by CSMC, NCTM, Achieve, College Board, MAA, ASA (February 2007).

numbers using words, numbers, pictures, and physical materials, and translate among representations.

5.2.A (revised part 1) Represent addition and subtraction of fractions and mixed numbers.

5.2.A (revised part 2) Translate among representations of fractions and mixed numbers.

Content Modification

7.1.G Solve multi-step contextual problems involving rational numbers and justify the solutions.

7.1.G (revised) Solve multi-step contextual problems involving rational numbers and linear equations and justify the solutions.

We want to state unequivocally that the ideas we suggest removing from the *Expectations* still belong in the standards document. They just belong under *Comments* rather than as part of the *Expectations.* For instance, the language in *Expectation 5.2.A*, discussing the types of representations of addition and subtraction of fractions and mixed numbers, belongs in the *Comments*. Additionally, this section should include examples of these types of representations and an explanation of the value of the different types. We see these as the "how" rather than the "what" and "when" of standards and see the right side of the page as the perfect place to include this kind of information.

Words such as "describe," "define" and "explain" are legitimate additions to the *Expectations* that address the conceptual aspect of mathematics and avoid the slide into instruction.

Recommendation 6: Create a standards document that is easily used by most people.

The February 29 standards document is easy to read and a vast improvement over the original standards. We appreciate that OSPI intends to offer it in multiple formats, including an online version with a hyperlinked glossary. This work would be premature now, but it is important that it not be lost.

High School

Because of time constraints, we have focused our review on the Algebra I, Geometry and Algebra II sequence. We justify this because the *Expectations* and *Comments* are equivalent in both sequences. At some point in time, more attention needs to be given to the content organization in the Mathematics I, II and III sequence.

High school has the same situation with language as do grades k–8 regarding clarity and precision. And, although the core mathematical topics for Algebra and Geometry are in the standards, there are issues regarding the content. There are a few instances of each of the following: missing content, mis-placed content, and content better served by being reorganized.

We were pleased to see the increase in rigor in the examples in the latest version of the standards. A few of the examples did not seem to be ideal illustrations, but we would want more time before we say anything specific with certainty.

Algebra II has additional complicating factors. We say more about this later.

Algebra I

The Algebra I standards are not as strong as those in K–8, although the most important part, the fundamental content of Algebra I is there. There are two areas of concern that are relatively easy to fix related to language and placement of Key topics. Two other areas, regarding the *Expectations* organization and the idea that the *Expectations* seem to have moved away from the conceptual aspect of algebra in favor of procedural manipulations, are more difficult to remedy.

Algebra has power because of the ease in which it lets us model contextual situations. Washington standards have always been exemplary on the conceptual side of mathematics and this shouldn't be lost. That said, the content of Algebra I is in OSPI's standards. For example, there is an extensive development of functions, including:

A1.2.G Describe relationships among the graphs of $f(x)$ and the transformations $f(x-h)$, $f(x)+k$, $cf(x)$, or combinations of these.

Linear equations and inequalities are done rigorously and thoroughly, for example:

A1.3.C Describe the relationship between the characteristics of the graph of a linear function and the parameters in a defining linear expression written in standard, slope-intercept, or point-slope form.

and:

A1.3.G Solve problems involving systems of two linear equations or inequalities, and interpret the solutions in the context of the problems.

The *Comments* include realistic problems that lend themselves to algebraic modeling. There is, however, a lack of explicit mention of the classical algebra problems, which are difficult to access without algebra.

As with the K–8 standards, there are a number of individual standards or comments have clarity issues related to the mathematics. For example, in the comments for the above-mentioned standard A1.3.G, we have:

> Solution methods include the use of algebra (substitution and elimination methods), graphs, and tables. Students are expected to represent solutions of systems of linear inequalities graphically.

We are concerned that algebraic solutions are not mentioned, since it is essential that students learn how to solve these equations and inequalities algebraically. Then they can associate their solutions to graphs and tables.

Similarly, in the comments for A1.7, the *Core Processes,* we have:

> Descriptions of solution processes, explanations, and justifications can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Earlier in this paper we discussed our opinion that specifying types of representations in the *Expectations* pushes into pedagogy. In Algebra I, we think it is even inappropriate in the *Comments* because the power of algebra is in its abstraction.

In addition to the necessary editing of individual standards, we see more intensive revision that is required. Sometimes this relates to the sequence of the *Expectations* as shown in A1.4.D–G, about quadratics. They start off well with:

> A1.4.D Write and solve quadratic equations to answer questions that arise in situations modeled by quadratic functions.

and end with the comment for A1.4.G that is explicit about the techniques that must be learned to find such solutions:

> Solution methods include the use of the quadratic formula, factoring, graphing, and completing the square.

The mathematics is all there, but the standards, comments and examples could to be reorganized in the following ways to improve the document. It could begin by defining basic skills needed, in part by rewriting the comments for A1.4.G as *Expectations*. It should then develop logically from skills to applications. Along the way, the algebraic forms of a quadratic expression, partially mentioned in the comments to A1.4.F, should be completed and moved into *Expectations*.

The second type of restructuring has to do with moving content from *Key Topics* to *Core Content.*

We see significant amounts of content review included in *Key Topics* in the Algebra I section, which we did not see in K–8. In the introduction to the standard, it says that *Key Topics* “might extend a previously learned skill, plant a seed for future development, or address a focused topic, such as scientific notation.” In this case, the Algebra I *Key Topics* cover much of the material taught in grade 7 and retaught in grade 8.

The A1.6 *Expectations* are about linear equations and inequalities. In addition to much of the material already having been taught in grade 8, the material is a better fit in section A1.3 on linear functions, equations and inequalities. It would be easy to integrate it there. Sometimes a standard in A1.6 can be included in A1.3 just by changing a word or two in A1.3. For example:

A1.6.A Write and solve linear equations and inequalities with one variable.

This is very similar to:

A1.3.A Write and solve equations and inequalities containing the absolute value of a linear expression in one variable, graph the solutions on the number line, and interpret the solutions in terms of distance.

A1.6.A could easily be incorporated into A1.3.A as:

A1.3.A Write and solve linear equations and inequalities and equations and inequalities containing the absolute value of a linear expression in one variable, graph the solutions on the number line, and interpret the solutions in terms of distance.

More will be said about the statistics that are included in Algebra I later.

Geometry

Most of the basic mathematical content for a Geometry course is included in the standards. Some of the content associated with formal Geometry is absent and there is a pedagogical bent to the standards that could be interpreted as investigation being the preferred approach to teaching Geometry.

But again, the majority of content that is central to Geometry is in the standards as illustrated by the following examples:

G.2.B Prove theorems about angles, including angles that arise from parallel lines intersected by a transversal.

G.2.F Prove and apply the Pythagorean theorem and its converse.

G.2.I Prove and apply properties of angles, arcs, chords, secants, and tangents of circles.

On the other hand, the *Expectations* never introduce other content that is fundamental to geometry: undefined terms, definitions, axioms and postulates.

Editing is needed for reasons beyond drafting well-crafted sentences. The *Core Processes* need to be revised so that the *Expectations* do not include language that distorts the standards into curriculum. The very first standard is pedagogy of the discovery sort incorporating “explorations and experiments” into the *Expectation*:

G1.A Use inductive reasoning based on explorations and experiments to make conjectures about geometric relationships.

G1.A (revised) Use inductive reasoning to make conjunctures about geometric relationships.

Inductive reasoning might well be tied to student explorations or experiments, but not always, as shown by the following example. Which of the following statements are true?

- All equilateral triangles are isosceles.
- Some isosceles triangles are right triangles and equilateral.
- No isosceles triangle is equilateral.

Known theorems from Geometry are being crowded by pedagogical *Expectations*. We have “make and prove” in G.2.C, G.2.E and G.3.D and “make and test” in G.2.J and G.4.B. These standards promote the philosophy that geometry should be learned by “discovery.” To us, again, that is *how,* not *what* or *when.*

There are other minor changes that we believe should be made. For example, the following standard and example:

G.2.M Analyze and describe cross sections formed by the intersection of planes and three-dimensional figures. (GGM.2.M)

Example:

Describe all the possible cross sections of a cube cut by a plane not parallel to a face.

This simple example of the cube illustrates how incredibly complex the content is in this standard. It is a rich classroom exercise, but it should not be a standard.

The placement of probability in the Geometry course will be discussed later.

Algebra II

Algebra II is now being asked to serve two different, quite legitimate purposes. First, there is traditional Algebra II, which is a content-dense offering that at least introduces such topics as complex numbers, polar coordinates, induction, inverse functions, matrices, conic sections, logarithms and series. It has historically been seen as a course for the mathematically elite and prepares students for calculus and math-intensive careers.

Within the past decade, there has been a national effort to require all students to take more mathematics, and this has usually meant Algebra II. Many people believe all students are best served by taking the course described above. Other people believe that a different sort of course, based on rigorous mathematics that is more directly useful in most people's lives, better serves students. Data, statistics, probability, discrete mathematics, optimization, fair divisions, and business or financial mathematics are some of the topics that are often included.

These two Algebra II courses have significant content in common, but they diverge.

When we examined OSPI's standards, we expected to see the standards for the traditional Algebra II course, aligned to the *Washington Readiness Standards.* Additionally, we thought we would see many of those standards identified as appropriate for the college bound. Instead we found most of the content the two types of Algebra would have in common. While we understand the rationale, at the very least, explanatory text should be provided.

Clear policy is needed to guide clear standards. Washington is moving toward requiring Algebra II or its equivalent for most students. It has a rare opportunity to clarify what Algebra II should be, if that should be the same for all students, and if not what it is that all students will learn.

Even given this situation, we have additional concerns about the Algebra II standards that need more than a strong edit to remedy. We have concerns about the *Performance Expectations,* the *Comments,* and the *Key Topics*, which are discussed below.

Performance Expectations and *Explanatory Comments and Examples*

We want to reiterate by example that there are many examples of strong *Expectations,* including:

A2.3.C Solve algebraic equations that involve the square root of a linear expression over the real numbers; identify extraneous solutions and explain how they arose.

A2.3.D Sketch the graphs of functions of the form $f(x) = a\sqrt{x-c} + d$ and cubic polynomial functions of the form $ax^3 + d$.

We want to note that the limits set in the comments related to the following *Expectations* are appropriate for general students, and to remind the reader that this is the lens we use for evaluation.

A2.1.E Add, subtract, multiply, divide, and simplify polynomials and rational algebraic expressions, and explain the procedures used.

- Multiplication should involve simple polynomials, such as a binomial by a trinomial.
- Division of polynomials is limited to polynomial divisors of degree one or two.
- Rational expressions should be limited to those with monomial or simple binomial denominators.

To give some examples of problems with the Algebra II standards, we start with A2.2 on exponential functions and equations. Logarithms and exponentials go together, and so including some work with logarithms in Algebra II seem appropriate given Algebra I includes so much work on exponentials. We would like to see *Expectations* similar to the following added to the standards:

Example: Know the properties of logarithms.

Example: Use logarithms to solve equations with exponentials in them.

Example: Recognize log functions as inverse functions of exponential functions.

There are many things that can be done in an Algebra II course. The specific inclusion of this one particular type of equation seems arbitrary and inexplicable:

> A2.3.F Solve equations of the form $\frac{f(x)}{g(x)} = k$, where *f*(*x*) and *g*(*x*) are polynomials of degree no greater than two.

There are other things that make the standards problematic. For example, complex numbers are mentioned only twice in the standards:

> A2.1.B Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.

> A2.3.B Solve quadratic equations with integer coefficients over real or complex numbers.

Nowhere in the standards is it suggested that the student must learn how to add, subtract, multiply and divide complex numbers. We understand why they would not be included in standards for all, but if complex numbers are to be used at all, we think these operations should be learned.

<u>Additional *Key Topics*</u>

As in Algebra I, review of previously learned material is included in *Key Topics*. Given that OSPI's standards are now structured to allow students to truly master topics, review topics do not warrant this kind of attention in *Key Topics*. As described in the introductory text of Washington *K–12 Mathematics Standards*, this is not a place to list topics that have previously been taught are only included in this section when they are to be extended. Since the "extension" in this case is much of the course content, we suggest it be integrated into the *Expectations*.

Comments on the data, statistics and probability section of Algebra II are included below.

Data, Statistics and Probability

To begin, there are some problems with the language of some of the *Expectations* for the Data, Statistics and Probability standards, that can easily be fixed. This is illustrated by the following:

A1.5.C Demonstrate that a bivariate data set can be modeled with a line of best fit, interpret the slope and *y*-intercept of the line in the context of the data, and use the line to interpolate and extrapolate in order to make and defend predictions.

A2.4.A Demonstrate that a bivariate data set can be modeled with an exponential or quadratic curve of best fit, approximate the curve of best fit, and use the curve to interpolate and extrapolate in order to make and defend predictions.

These two standards just need to make clear that there are different ways to model bivariate data.

More important, there are some complex issues related to Data, Statistics and Probability that plague educators and school systems across the country.

One is what kind of content is appropriate, and the other is how to add that content to already-full course syllabuses.

Data and statistics are part of life. On television you hear polling results saying 32 percent of a particular subgroup supports a certain candidate with a 3 percent confidence interval. In the newspaper you read that the mean price for a house has dropped $30,000. So, what is it that students need to know to make sense of the world, and how well do they need to know it?

Students do not learn the mathematics that under lies some of the statistics in OSPI's standards for Algebra I. The basics of items such as standard deviation, line of best fit and correlation coefficients are mathematically inaccessible to high school freshmen. They can understand the concept and use the results, but they are "black box" items found using a calculator.

Some people are troubled by this and think students should not engage in mathematics they can't "do." For example, a student never learn the real definition of line of best fit because he or she does not pursue higher mathematics. The question is, would we prefer students who have no experience with this concept or students who can use the concept but without complete understanding?

Then there is the question of where the content fits in the curriculum. Most Algebra and Geometry textbooks do include some Data, Statistics, and Probability content, but many teachers would agree that is it is difficult to cover everything in a text. OSPI's standards have more than is usually seen but not more than can be justified given the topic's importance. Given that one needs the information in today's world, how does one fit it in?

Strategic Teaching Recommendations

The *Washington State K–12 Mathematics Standards* are a vast improvement over the standards that were in place a year ago. They are worth taking the additional time and effort to make them excellent, truly world class. To that end, we recommend the following:

1. An exemplar review on the OSPI February standards a) K–8 and b) 9–12 similar to last year's comparison to other states, countries and national frameworks using the nine criteria to provide external validation that these are the best standards. In order to compress the timeline and keep the standards-writing process as close to schedule as possible, we suggest fewer grade levels and fewer documents.

2. Substantive edit for grades K–8. The content is very good; language is almost ready. These standards are so close that work could be completely very quickly.

3. A revision of the high school standards. The core content of the subjects is in the document and many of the examples are excellent. The language needs to be tightened and there is some more work to be done on the content. This means it will take slightly longer than the grade K–8 work.