Commentary

Precision in the Teaching, Learning, and Communication of Elementary School Mathematics: A Reply to Wilson's "Elementary School Mathematics Priorities"

Carolyn Maher, EdD
Professor II
Graduate School of Education
Rutgers University
New Brunswick, NJ

Keith Weber, PhD
Assistant Professor
Graduate School of Education
Rutgers University
New Brunswick, NJ

Goals of elementary school mathematics

In "Elementary School Mathematics Priorities," Wilson (2009 [this issue]) presents a list of five core concepts that students should master in elementary school so that they can succeed in algebra. As researchers in mathematics education, we enthusiastically endorse Wilson's recommendations.

Learning algebra is key to further study of mathematics. If students are unable to pass an algebra course, they will likely not graduate from college, perhaps not even high school. This can limit students' access to high-paying jobs. Hence, students' success in algebra can strongly impact their economic well-being.

For these reasons, preparing all students to succeed in algebra must be a primary goal of mathematics classes; in fact, we suggest that early algebra learning begin in the elementary grades. Students are unlikely to make progress in understanding the ideas of algebra if they have not mastered the skills of basic arithmetic, including: (a) understanding our base ten number system, (b) instantly recalling facts about single digit arithmetic, and (c) performing the basic arithmetic operations on whole numbers, fractions, and decimals.

Wilson emphasizes that these concepts, facts, and algorithms should be learned with understanding and through mathematical reasoning. On all of these points, we strongly concur.

As mathematics educators, we argue that there are other important mathematical competences that students should also develop in addition to the skills that Wilson (2009) lists. Elementary school students should have experience representing mathematical concepts in different ways, justifying why their mathematical solutions are correct, and communicating their ideas to others. Wilson states that his list of skills that must be mastered in elementary school is not exhaustive, and we concur.

That said, the following points are intended for clarification, rather than contention.

Learning elementary mathematics

We turn our attention to issues about the role that precision should play in elementary mathematics learning. Wilson (2009) argues that precision is fundamental for doing mathematics, stating that "precision, lack of ambiguity and hidden assumptions, and mathematical reasoning are the fundamental defining principles of mathematics and it is difficult to adequately emphasize their importance." He returns to the matter of precision numerous times in his article, at one point claiming that, "students must use the precise terms, operations, and symbols of mathematics."

We agree with these sentiments insofar as they pertain to standard outcomes of elementary mathematics education; as students move through elementary school, they should make progress in the correct use of precise terms, operations, and symbols of arithmetic.

We question whether insistence on precision is necessary, or even desirable; when students are first being introduced new mathematical concepts as well as when they explore and invent their own personal notations during process of mathematical learning and problem solving.

Wilson (2009), on the other hand, believes this point and claims, "the proper use of language is essential to the learning process." He goes on to warn that, "the meaning of terms, operations, and symbols must be completely unambiguous or communication is lost and meaning slips away."

If students are using terms, operations, or symbols that have a meaning in standard mathematics, we expect them to be using those terms correctly. Certainly, the incorrect use of mathematical terminology can develop or reinforce misconceptions about mathematical ideas and stymie future learning.

However, to this point, we offer research that depicts the process by which students build meaning as often messy and imprecise. When children are first introduced to a new mathematical idea, they often invent personal operations and notations to use in their mathematical work.

The process of exploring mathematical activities is a first step for students to develop personal meaning that can be represented a variety of ways. Our work has shown that this is a prerequisite for students building deep conceptual understanding of the mathematics that they are doing. Hence, this process is integral, in that it enables the students to give meaning to the standard mathematical terms, operations, and symbols that they will utilize in the future.

By encouraging students to invent operations and notations and to share these ideas with their classmates, we contend that the terms, operations, and symbols that students use will not be completely unambiguous. Wilson (2009) fears that this potential ambiguity will cause communication to be lost and meaning to slip away.

We disagree with this assertion. In our work, we anecdotally have found that students are capable of effectively communicating informal mathematical ideas to each other using novel representations.

Furthermore, the process of resolving natural differences in such representations, can provide a rich learning opportunities for students. Asking students to explain the meanings behind their mathematical terms and procedures affords them the opportunity to think more deeply about the underlying mathematical concepts, clarify what their symbols represent and defend why their actions make sense.

To avoid misinterpretation, we acknowledge that the use of standard mathematical terms, operations, and symbols is a necessary strategy for learning and doing mathematics as it makes communication about complex ideas possible, thereby facilitating understanding. Yet, we contend that insisting students use these terms, operations, and symbols before they are meaningful to them will actually be an impediment to understanding and communication of ideas. That is, asking students to apply terms that they do not yet understand requires them to make assertions that do not make sense to them.

To further our position on precision vis-à-vis that of Wilson (2009), we also find discomfort with the demand for precision in the posing of problems to children. At several points in his article, Wilson mandates that there be no hidden assumptions in a mathematics problem. He asserts, “if a problem is not well-defined with a unique set of solutions, it is not a mathematics problem. There can be no hidden assumptions in a real mathematics problem.”

Is this claim plausible? There are numerous conventional problems in school mathematics that clearly rely on hidden assumptions. When children are asked the sum of the degrees in a triangle, it is assumed that they are working within the Euclidean geometry. When students are asked why a squared number cannot be negative, it is assumed that they are working with the real number system.

Specifying when it would be appropriate to use a particular estimation strategy does not have a unique set of solutions, but this would be a productive conversation to have in an elementary mathematics classroom. Many word problems rely on an implicit shared understanding of how the world works.

Posing problems with hidden assumptions is not only unavoidable, but also advantageous in some situations. One of the most robust findings in mathematics education research is that students enter classrooms with mathematics assumptions that are incorrect. Further, giving students mathematical definitions and requiring them to be precise does not prevent their incorrect assumptions from influencing their reasoning (e.g., Tall & Vinner, 1981).

Effective teaching requires making students’ meanings public thereby having the opportunity to address their misconceptions directly. If students obtain different answers to the same mathematical problem, they naturally will be motivated to revisit and explore the assumptions that led to this apparent contradiction. Discussions of this type have been shown to create powerful learning opportunities (Maher & Martino 1996; Weber, Maher, Powell, & Lee, 2008).

Support for our views

Reform-oriented mathematics curricula ask students to invent operations and notations to solve problems that may rely on hidden assumptions. Several large studies have compared the learning outcomes of those who learned using reform-oriented and traditional curricula (e.g., Senk & Thompson, 2003, 2006).

In general, the students taught using the reform-oriented curricula developed a greater conceptual understanding of algebra without loss of procedural skill. We are not claiming that all reform-oriented curricula are without fault or cannot be improved, but these studies do demonstrate that some of these curricula are useful in helping students succeed in algebra.

Our beliefs are largely shaped by our own experience with creating informal mathematical learning environments. In these environments, we ask students to collaboratively work on challenging and open-ended mathematical tasks. When doing so, students spontaneously generate a wide array of notations and operations to solve our problems, justify their solutions, and communicate with each other.

The results of our study show that young students were able to solve very difficult problems (e.g., Maher & Martino, 1996). Further, although their initial explanations were expressed using their own terminology, which was sometimes imprecise, they often were able to later express their ideas using standard mathematical notation appropriately (Maher, 2005; Uptegrove & Maher, 2004). Students who participated in our program showed improvements in their standardized test scores (Maher, 1991) and the mathematical dispositions that they developed led them to not only succeed in algebra, but graduate from respected universities (Francisco, 2005).

Author Biographies

Carolyn Maher teaches in the department of learning and teaching at Rutgers University. She is also the director of the Robert B. Davis Institute for Learning at Rutgers and the editor of the *Journal of Mathematical Behavior*. Her research interest deals with reasoning, justification, and proofmaking.

Keith Weber is an assistant professor in the Rutgers University Department of Learning and Teaching and is chair of the mathematics education specialty for the PhD in education at Rutgers. His research interest also deals with reasoning, justification, and proofmaking.

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