# Arithmetic, Division, Calculus III, and Beyond

by

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## Introduction

There is a general belief among many mathematicians that facility with arithmetic is important for learning advanced mathematics despite the fact that little arithmetic is used in most advanced mathematics courses. To check this out, I gave a ten question arithmetic test to strong students. I found that those who missed 3 or more questions did significantly worse in my course than those who missed fewer. I then looked closer at the long division problem because there were students who did not know how to even approach the problem. By itself, cluelessness for long division did not correlate with grades as well as the test taken as a whole. However, I followed up on the class two years later and found that one-third of the division-clueless students had been on academic probation. The probability that this could happen at random was only 1%.

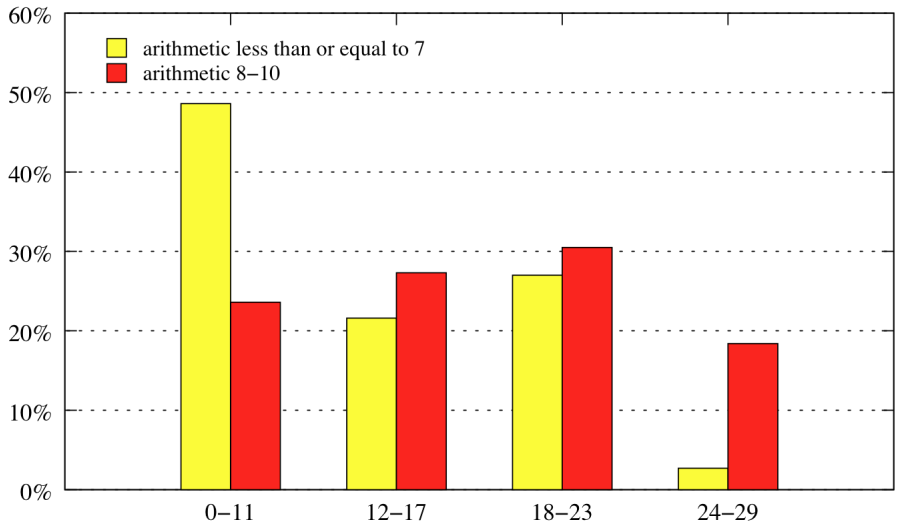
## The Students

In the fall of 2007 I taught *Calculus III* to over 200 students. These students were mostly freshmen. Johns Hopkins University is a fairly selective school to get into and *Calculus III* is a course only taken by rather serious and good students. *Calculus III* goes by various names: *Vector Calculus, Multivariable Calculus, Calculus of Several Variables.* There is a lot of material in the course and it goes very fast. However, there is little use of basic arithmetic. *Calculus III* obviously comes after *Calculus I & II*, and is more or less coequal with *Linear Algebra* and *Differential Equations*, the main courses most of our engineers and physical science majors take.

## Arithmetic

On the first day of class in 2007 I gave a ten question basic arithmetic test, which can be found at http://www.math.jhu.edu/~wsw/ED/arith.pdf. I divided the students into two groups, those who got 8-10 of the problems correct and those who got 0-7 of the problems correct. When the course was over I looked at the final exam and divided students up into four groups based on their score for the final. The maximum score was 29. For each of these four groups I have graphed the percentage of each type of student (0-7 or 8-10) in that group. See the graph in Figure 1. The students who dropped the course were assigned the bottom group on the final exam because this is a required course for nearly everyone who takes it.

Figure 1: Arithmetic test scores and final exam scores



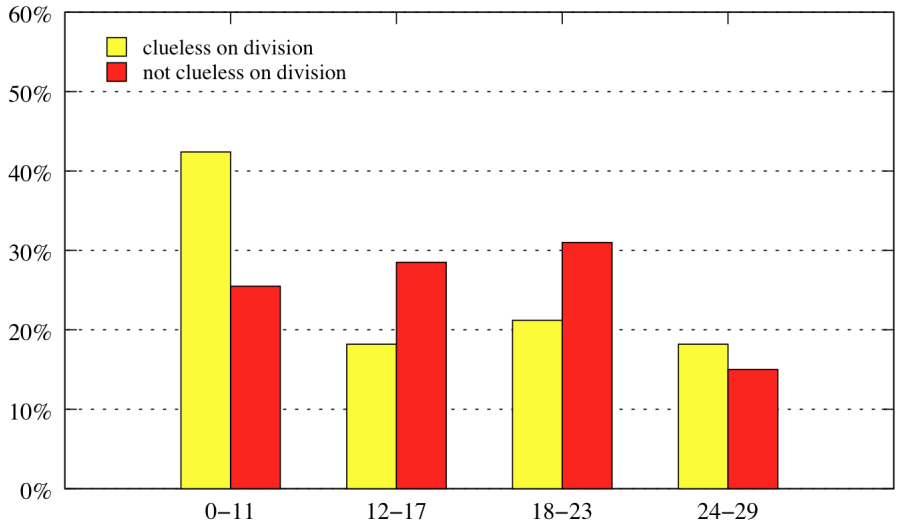

This graph is rather telling. The top achievers have a good grasp of arithmetic and without a good grasp it is highly unlikely that a student will be among the high achievers. At the other end, though, we see students without a good grasp of arithmetic at the bottom end of the spectrum at twice the rate of those who can do arithmetic.

## Long Division

While grading this exam I noticed that a number of students didn't even try to solve the long division problem, "divide 51.072 by 0.56," or, if they did try, they seemed to be unfamiliar with the usual symbol for long division. The bottom line was that they didn't even know how to get this problem wrong.

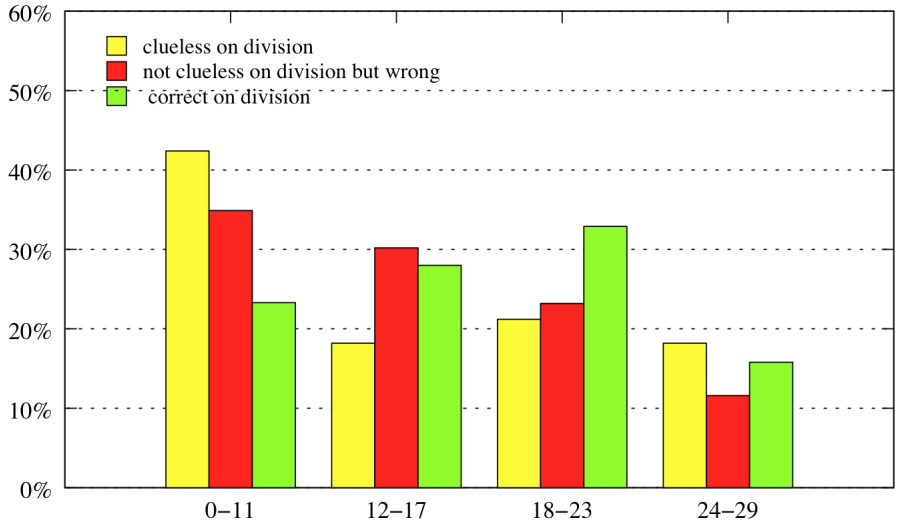
In fact, 33 of my students fell into the "clueless" about division category. This gave me the graph in Figure 2 of the above sort.

Figure 2: Division and final exam scores




This certainly wasn't much support for the hypothesis that long division mattered! Not everyone who knows how to do long division gets the right answer, so I put that into Figure 3.

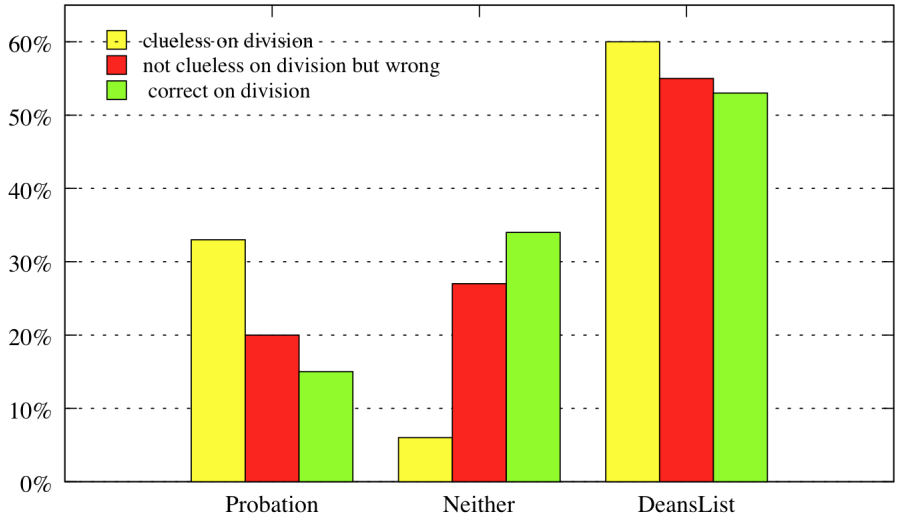
Figure 3: Division and final exam scores




On the low end, it looks like there is a real difference between the clueless and the correct, but this still isn't very strong. No doubt students who have been deprived of long division in their education must be quite strong problem solvers to be able to gain admission to Johns Hopkins University and make it to *Calculus III*.

I was still curious about this in the fall of 2009 and a departmental staff member, Sabrina Raymond, tracked down all the students from my fall 2007 class, which results in the graph of Figure 4.

Figure 4: Division, Percentages Deans List and Probation




Now we see something interesting: a bipolar distribution for the clueless. Eleven of our 33 division-clueless students had been (or were) on probation. I then checked with my local statistician, Daniel Naiman. With 40 students out of my original 236 managing to achieve probation status, the chance that 11 or more out of a random 33 being on probation is 1%.

## Conclusion

Although we only see correlations and not causation, my personal conclusion is that students who are not expected to master basic arithmetic, particularly long division, have been needlessly handicapped by their schooling. Those without a good grasp of arithmetic appear not to achieve at a high level in college mathematics and are over represented at the low end. Long division is a somewhat different story. Students who are clueless about long division but who can get into Johns Hopkins University and *Calculus III* are presumably quite bright and could have mastered long division if given a chance. They somehow seem to either survive quite well, working around their (unnecessary) handicap, or suffer seriously.