ARITHMETIC VERSUS THE ELITE

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Abstract. This is a more detailed analysis of an arithmetic test I gave to my multi-variable calculus students in the fall of 2007.

1. Introduction

This paper came about from a conversation I had with a well respected mathematician who was heavily involved with K-12 mathematics education. He was saying that there were places in the 5-th grade that calculators were okay. I asked him what kinds of problems were appropriate for calculators there. He showed me the addition of fractions with rather non-standard denominators and said a four function calculator would be appropriate. I suggested that students still needed practice with the multiplications required or they wouldn't be able to do them in college. He said that less than 1% of college students would not be able to do a multiplication. I had given an arithmetic test a few years back so I went and checked my multiplication problem. He was a little off on his estimate.

In the fall of 2007 I gave a 10 question arithmetic test to my 229 Calculus III (multi-variable calculus) students on the first day of class. Among other things, this means they already had credit for a full year of Calculus. The vast majority of these students were freshmen and were not about to question what happens on the first day of class in college. They had plenty of time. The average math SAT score was about 740.

What follows is a question by question description of the test results, obviously given without the use of calculators

The test had previously been written about and posted at:

`http://www.math.jhu.edu/∼wsw/ED/arithgeomstudy08.pdf`

2. RESULTS

1. Multiply 5.78 by 0.390. (answer 2.2542) 23% of my students missed this, i.e. 52.

Although this isn't just whole number multiplication, it is quite a bit higher than 1%, and these are not average students. I shudder to think.

2. Subtract 5.897 from 68.25. (answer 62.353) 8% of my students missed this, i.e. 18.

3. Divide 3 1/6 by 7/18. (answer 57/7) 10% of my students missed this, i.e. 22.

4. 70 is what percent of 250? (answer 28%) 16% of my students missed this, i.e. 37.

5. Find 25% of 340. (answer 85) 1% of my students missed this, i.e. 3.

Notice the dramatically different results on the two percentage problems. Even the bad one is vastly superior to what you read in the newspapers. These are good students.

6. Find the sum 5/6 + 4/5. (answer 49/30) 3% of my students missed this, i.e. 7.

7. Subtract 3 1/9 from 5 2/3. (answer 23/9) 11% of my students missed this, i.e. 26.

8. Find the sum: 3.29 + 78.985 + 18. (answer 100.275) 8% of my students missed this, i.e. 19.

9. Divide 51.072 by 0.56. (answer 91.2) 33% of my students missed this, i.e. 76.

This is the only problem I really followed up on in the past. 14%, or 33 of my students were completely clueless on this problem. The other 43 had been taught, but didn't get the right answer. 11 of those 43 just misplaced the decimal, a serious problem in its own right. Of the 33 clueless, 11 ended up on probation. I've written about them in:

`http://www.math.jhu.edu/∼wsw/ED/arith3.pdf`

10. Add 2/3 and 1/2, and divide the result by 5/12. (answer 14/5) 5% of my students missed this, i.e. 11.

3. Conclusions

One question to ask here is about stamina, or just the general ability to follow through and finish. For example, there are 4 questions about fractions on the exam. The highest miss rate (for problem #7) is 11%, but 52 students, or 23%, missed at least one fraction question. Some would argue that 11% is low, so this indicates that my students know how to deal with fractions. I'm not so sanguine. These are very good students, and if they know fractions but are just really careless, that is just as much of a problem as not knowing fractions.

There were also 4 questions using decimals. The highest miss rate (for problem # 9) was 33%, but 111 students, or 48%, missed at least one. That's half of my reasonably elite students who couldn't get all four arithmetic operations correct with decimals in one sitting. I don't think they get decimals.

Seven of these problems were each missed by 8% or more of my students and 69 students, or 30%, missed more than 1 problem.

These are high achieving, highly motivated students (remember the 740 average SAT math score). These are disturbing numbers for them, but I suspect the numbers are much much worse among college freshmen with an average math SAT of 582, and, from Table 145 of *Digest of Education Statistics 2009* we see that the average SAT score for the intended college major of engineering is 582 in 2008-2009.

Anyway, there is no real purpose to this paper except as a resource for me. It does suggest, very strongly, to me, that we have lost the pro-arithmetic war. This is a revelation to me and it calls into question what I will do next year with my big service course. I now feel compelled to assume that they are chronically accident prone or they really are arithmetically handicapped. It isn't clear that there is a difference. The question remains, how can I teach serious college level mathematics to students who are ill-prepared?

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