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The Faulty Logic of the 'Math Wars'

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There is a great progressive tradition in American thought that urges us not to look for the aims of education beyond education itself. Teaching and learning should not be conceived as merely instrumental affairs; the goal of education is rather to awaken individuals' capacities for independent thought. Or, in the words of the great progressivist John Dewey, the goal of education "is to enable individuals to continue their education."

This vision of the educational enterprise is a noble one. It doesn't follow, however, that it is always clear how to make use of its insights. If we are to apply progressive ideals appropriately to a given discipline, we need to equip ourselves with a good understanding of what thinking in that discipline is like. This is often a surprisingly difficult task. For a vivid illustration of the challenges, we can turn to raging debates about K-12 mathematics education that get referred to as the "math wars" and that seem particularly pertinent now that most of the United States is making a transition to Common Core State Standards in mathematics.

At stake in the math wars is the value of a "reform" strategy for teaching math that, over the past 25 years, has taken American schools by storm. Today the emphasis of most math instruction is on — to use the new lingo — numerical reasoning. This is in contrast with a more traditional focus on understanding and mastery of the most efficient mathematical algorithms.

A mathematical algorithm is a procedure for performing a computation. At the heart of the discipline of mathematics is a set of the most efficient — and most elegant and powerful — algorithms for specific operations. The most efficient algorithm for addition, for instance, involves stacking numbers to be added with their place values aligned, successively adding single digits beginning with the ones place column, and "carrying" any extra place values leftward.

What is striking about reform math is that the standard algorithms are either de-emphasized to students or withheld from them entirely. In one widely used and very representative math program — TERC Investigations — second grade students are repeatedly given specific addition problems and asked to explore a variety of procedures for arriving at a solution. The standard algorithm is absent from the procedures they are offered. Students in this program don't encounter the standard algorithm until fourth grade, and even then they are not asked to regard it as a privileged method.

The battle over math education is often conceived as a referendum on progressive ideals, with

those on the reform side as the clear winners. This is reflected, for instance, in the terms that reformists employ in defending their preferred programs. The staunchest supporters of reform math are math teachers and faculty at schools of education. While some of these individuals maintain that the standard algorithms are simply too hard for many students, most take the following, more plausible tack. They insist that the point of math classes should be to get children to reason independently, and in their own styles, about numbers and numerical concepts. The standard algorithms should be avoided because, reformists claim, mastering them is a merely mechanical exercise that threatens individual growth. The idea is that competence with algorithms can be substituted for by the use of calculators, and reformists often call for training students in the use of calculators as early as first or second grade.

Reform math has some serious detractors. It comes under fierce attack from college teachers of mathematics, for instance, who argue that it fails to prepare students for studies in STEM (science, technology, engineering and math) fields. These professors maintain that college-level work requires ready and effortless competence with the standard algorithms and that the student who needs to ponder fractions — or is dependent on a calculator — is simply not prepared for college math. They express outrage and bafflement that so much American math education policy is set by people with no special knowledge of the discipline.

Even if we accept the validity of their position, it is possible to hear it in an anti-progressivist register. Math professors may sound as though they are simply advancing a claim about how, for college math, students need a mechanical skill that, while important for advanced calculations, has nothing to do with thinking for oneself.

It is easy to see why the mantle of progressivism is often taken to belong to advocates of reform math. But it doesn't follow that this take on the math wars is correct. We could make a powerful case for putting the progressivist shoe on the other foot if we could show that reformists are wrong to deny that algorithm-based calculation involves an important kind of thinking.

What seems to speak for denying this? To begin with, it is true that algorithm-based math is not creative reasoning. Yet the same is true of many disciplines that have good claims to be taught in our schools. Children need to master bodies of fact, and not merely reason independently, in, for instance, biology and history. Does it follow that in offering these subjects schools are stunting their students' growth and preventing them from thinking for themselves? There are admittedly reform movements in education that call for de-emphasizing the factual content of subjects like biology and history and instead stressing special kinds of reasoning. But it's not clear that these trends are defensible. They only seem laudable if we assume that facts don't contribute to a person's grasp of the logical space in which reason operates.

The American philosopher Wilfrid Sellars was challenging this assumption when he spoke of "material inferences." Sellars was interested in inferences that we can only recognize as valid if we possess certain bits of factual knowledge. Consider, for instance, the following stretch of reasoning: "It is raining; if I go outside, I'll get wet." It seems reasonable to say not only that this is a valid inference but also that its validity is apparent only to those of us who know that rain

gets a person wet. If we make room for such material inferences, we will be inclined to reject the view that individuals can reason well without any substantial knowledge of, say, the natural world and human affairs. We will also be inclined to regard the specifically factual content of subjects such as biology and history as integral to a progressive education.

These remarks might seem to underestimate the strength of the reformist argument against "preparatory" or traditional math. The reformist's case rests on an understanding of the capacities valued by mathematicians as merely mechanical skills that require no true thought. The idea is that when we apply standard algorithms we are exploiting 'inner mechanisms' that enable us to simply churn out correct results. We are thus at bottom doing nothing more than serving as sorts of "human calculators."

There are philosophical precedents for this understanding of mathematical algorithms. Some philosophers envision the capacity to apply an algorithm as a sort of psychological mechanism that merely causally produces the right answer. They in effect invite us to see mastering an algorithm as like hitching ourselves to a mechanical rail and having it pull us along.

This mechanical image of calculation is the target of a number of philosophical critiques. Arguably the most famous of these is in Wittgenstein's later writings. Wittgenstein suggests that analogies between mathematical computations and mechanical processes only seem appealing if we overlook the fact that real machines have parts that bend and melt and are invariably subject to breakdown. If, however, we keep this fact in mind, it seems clear that a psychological mechanism cannot ground my confidence that I am getting things right. That is, it seems clear that, in order to be justified in believing that I have mastered an algorithm, I require a type of mental activity that isn't simply causally generated. That, according to Wittgenstein, is why it is wrong to understand algorithm-based calculations as expressions of nothing more than "mental mechanisms." Far from being genuinely mechanical, such calculations involve a distinctive kind of thought.

That the use of standard algorithms isn't merely mechanical is not by itself a reason to teach them. It is important to teach them because, as we already noted, they are also the most elegant and powerful methods for specific operations. This means that they are our best representations of connections among mathematical concepts. Math instruction that does not teach both that these algorithms work and why they do is denying students insight into the very discipline it is supposed to be about.

(Reformists sometimes try to claim as their own the idea that good math instruction shows students why, and not just that, algorithms work. This is an excellent pedagogical precept, but it is not the invention of fans of reform math. Although every decade has its bad textbooks, anyone who takes the time to look at a range of math books from the 1960s, 70s or 80s will see that it is a myth that traditional math programs routinely overlooked the importance of thoughtful pedagogy and taught by rote.)

As long as algorithm use is understood as a merely mechanical affair, it seems obvious that

reformists are the true progressivists. But if we reject this understanding, and reflect on the centrality to mathematical thought of the standard algorithms, things look very different. Now it seems clear that champions of reform math are wrong to invoke progressive ideals on behalf of de-emphasizing these algorithms. By the same token, it seems clear that champions of preparatory math have good claims to be faithful to those ideals.

There is a moral here for progressive education that reaches beyond the case of math. Even if we sympathize with progressivists in wanting schools to foster independence of mind, we shouldn't assume that it is obvious how best to do this. Original thought ranges over many different domains, and it imposes divergent demands as it does so. Just as there is good reason to believe that in biology and history such thought requires significant factual knowledge, there is good reason to believe that in mathematics it requires understanding of and facility with the standard algorithms. Indeed there is also good reason to believe that when we examine further areas of discourse we will come across yet further complexities. The upshot is that it would be naïve to assume that we can somehow promote original thinking in specific areas simply by calling for subject-related creative reasoning. If we are to be good progressivists, we cannot be shy about calling for rigorous discipline and training.

The preceding reflections do more than just speak for re-evaluating the progressive credentials of traditional, algorithm-involving math. They also position us to make sense of the idea, which is as old as Plato, that mathematics is an exalted form of intellectual exercise. However perplexing this idea appears against the backdrop of the sort of mechanical picture favored by reformists, it seems entirely plausible once we recognize that mathematics demands a distinctive kind of thought.

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