A LITTLE STARTER TEST FOR CALCULUS 2

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1. Introduction

On the first day of class, Fall, 2013, I gave my *Calculus II for the Biological and Social Sciences* class a short exam of eight questions and gave them the full 50 minutes.

104 students were there that first day and took the exam, including at least one student who was supposed to be in the chemistry class that met in a room on a different floor.

88 students finished the course.

73 of those 88 had been there and taken the test on the first day of class. I will mainly focus on these students.

I had the SAT math scores for the freshmen class, but only 62 of the students who took the test on the first day were freshmen and only 47 of those had taken the SAT math test. That 47 gets into the realm of small numbers.

To get into this class, a student, in principle, had to have a 5 on the Calculus AP AB test or a 3 or 4 on the BC. Motivated students found loopholes and managed to get in anyway.

Either admissions and/or our screening have improved because I had SAT math scores for the same course for the fall of 2002 and the class average was 704, but this year it was 726, quite a difference, and I assure you, it could be felt in the class. (This is, of course, just for the freshmen.)

I have divided the class of 73 students of interest into 4 parts, which isn't that easy because 73 is a prime. I did this using the scores on the final exam because they are more fine grained than the letter grades for the course. Part 1 is the bottom 17 students on the exam, part 2 the next 20 (4 students had the same score at the boundary), part 3 the next 18, and part 4 the top 18 on the final.

Of the 104 students, they break down as follows: 82 Freshmen, 15 Sophomores, 5 Juniors, and 2 Seniors.

Of the 73 students of interest, there were 62 Freshmen, 7 Sophomores, 3 Juniors, and 1 Senior.

No calculators were allowed.

2. ARITHMETIC RESULTS

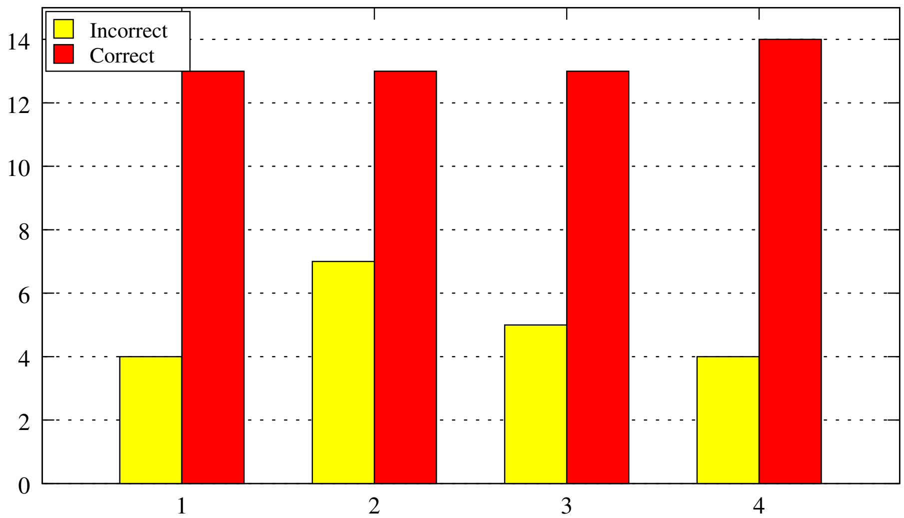
1. *Add*

$$\begin{array}{r} 1029 \\ 3847 \\ 5610 \\ 2938 \\ \underline{4756} \end{array}$$

Analysis: I was pleasantly surprised by the results for this problem. A few years ago I asked a big class if they had ever added up 5 4-digit numbers and they looked at me like I was an idiot and indicated that very few had. That is obviously not the case as most students just did this. Of course, getting the correct answer wasn't so straightforward, but at least they knew how to do it. 75 just did it correctly using what appeared to be the standard algorithm. 28 just did it incorrectly using what appeared to be the standard algorithm. 2 other students did it correctly, but without using the standard algorithm but adding two numbers together, then adding the next to that, etc. Neither stayed with the course.

Of the 73 students of interest, they all did it using the standard algorithm, but 20 of them got incorrect answers. The uselessness of this information is illustrated by Figure 1.

Figure 1: Number of students



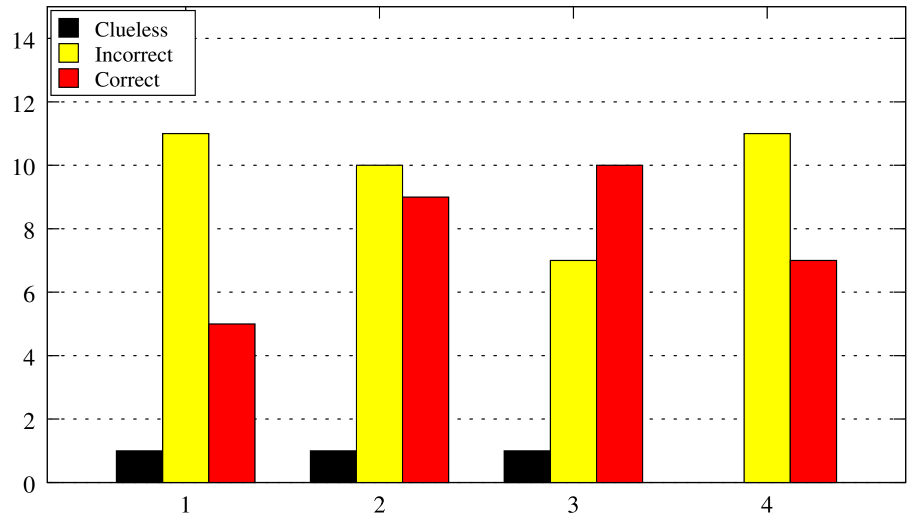
2. *Multiply:*

$$\begin{array}{r} 2938 \\ \underline{4756} \end{array}$$

Analysis: Again I was pleasantly surprised. Most students just did this using the standard algorithm. In all of the time of my involvement with K-12 math education, I have seen no indication that anyone ever does a multiplication of a 4-digit number by another 4-digit number. My students took this in stride, suggesting that there are pockets of teaching out there or that it really is enough to teach 2-digit by 2-digit multiplication. 50 got the right answer and 45 didn't. 8 students were clueless and did weird things and didn't get the answer. 1 student used the lattice method and got the right answer (but dropped the course).

When we move to our 73 students who finished the class, there were 3 who used non-standard methods, if anything, and got the problem wrong. 39 got the problem wrong, but presumably knew how to do it. 31 got it right using the standard algorithm.

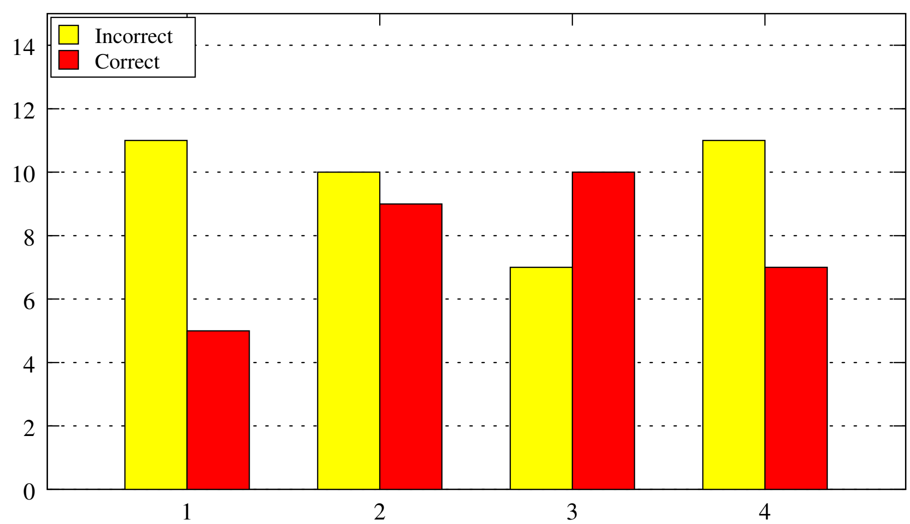
Figure 2: Number of students



The graph, again, tells us little although it does look like people who got the right answer were less likely to be at the bottom of the class.

It is sort of pointless keeping the clueless separate from those who just got it wrong, although they are the more interesting part of the population. Merging the clueless with the incorrect we have Figure 2a, which doesn't show us much more.

Figure 2a: Number of students

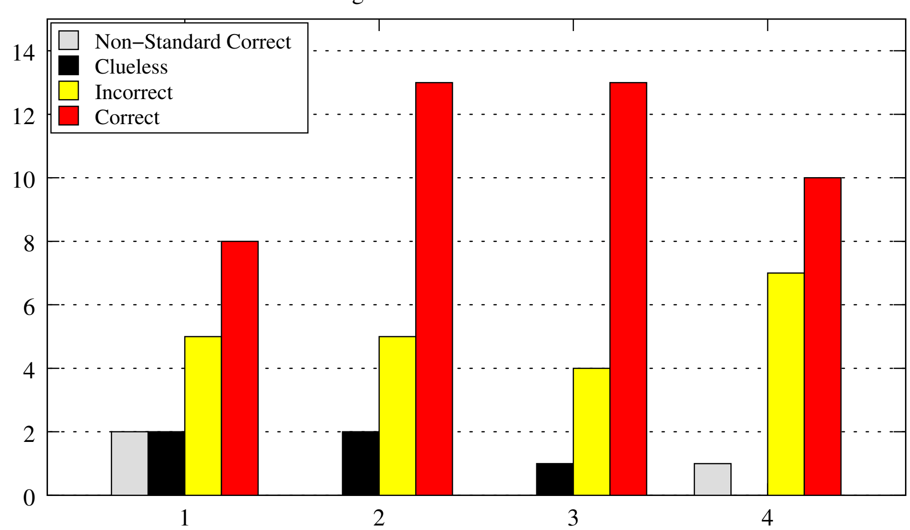


3. *Divide 585,331 by 857.*

Analysis: You can find a study on my website where a few years ago I gave much more advanced students a long division problem that had decimals in it. 1/7 of those students where truly clueless about how to do the problem. Here there were 14 clueless students, very close to 1/7 (more than 1/8), and another 4 students who used alternative techniques to get the correct answer. On the decimal problem, they might not have succeeded. For those who just plugged ahead with the standard algorithm, 60 got the right answer and 26 didn't.

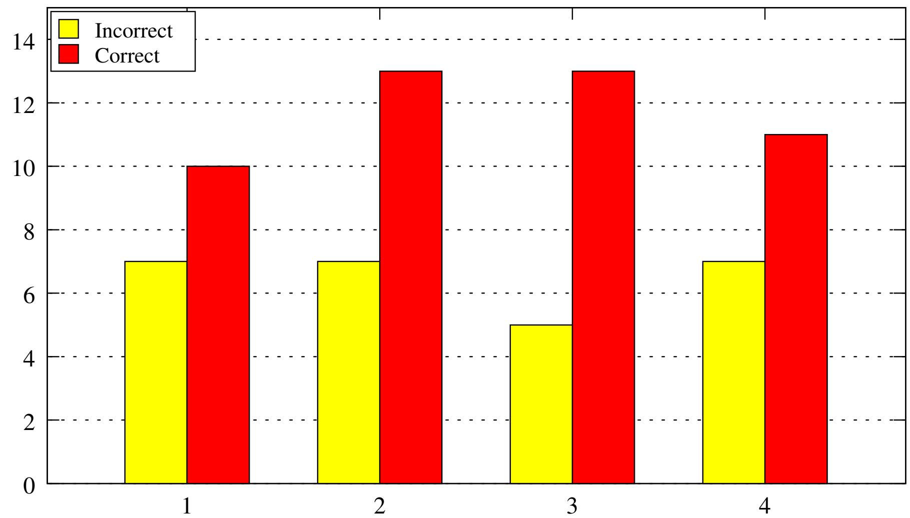
For our 73 special students, 5 were clueless, 21 knew how to do it with the standard algorithm but got it wrong, 44 did it right. There was a new category though as 3 students used non-standard techniques and got it right.

Figure 3: Number of students



I'm inclined to want to put those who use non-standard techniques in the same category as those who are clueless, but since they all got the right answer, I feel constrained. If I merge clueless and incorrect, and I merge non-standard and correct, we get Figure 3a.

Figure 3a: Number of students



Arithmetic Summary:

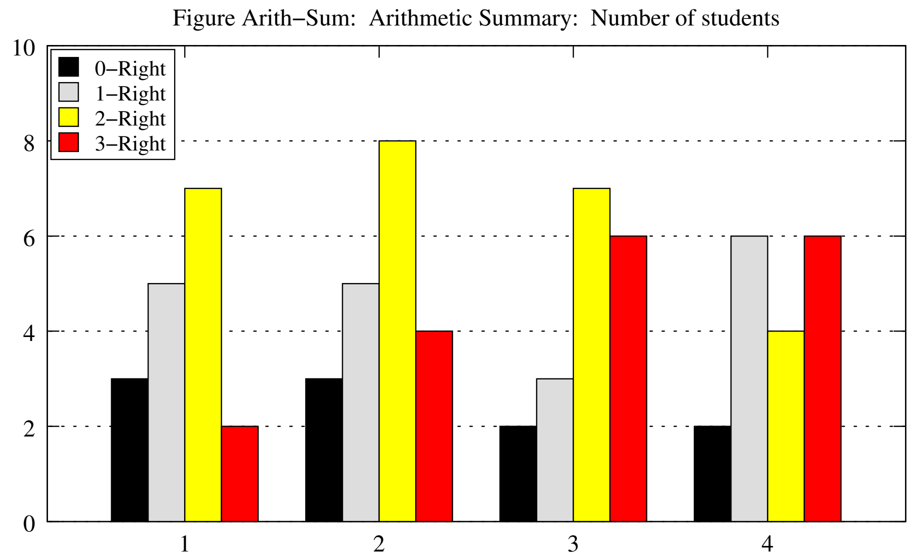
I'm a little surprised that more students got the division problem correct than the multiplication problem.

Of the 104 students, 27 got all 3 of the above correct using the standard algorithms. Another 32 just missed one. 20 missed two and 4 missed all three using the standard algorithms.

17 students were clueless on at least 1 of the 3. 7 students did one non-standard approach to one of the problems and got it right.

For my 73 students of interest, 1 student was clueless on two problems and got the third correct. 1 student was clueless on one problem and missed the other two. 5 students got one right, one wrong, and were clueless on the third. 3 got all 3 wrong. 19 got 1 right and 2 wrong. 26 got 2 right and 1 wrong. 18 got all 3 right.

I counted the clueless as -1, incorrect as 0, and correct, even if non-standard, as 1, but didn't let anyone get a score below zero.



This doesn't seem to be that informative except to suggest that you are better off if you work these problems correctly consistently. I'm not going to do the deep dark statistics on that, but if someone does, please let me know.

3. A little algebra

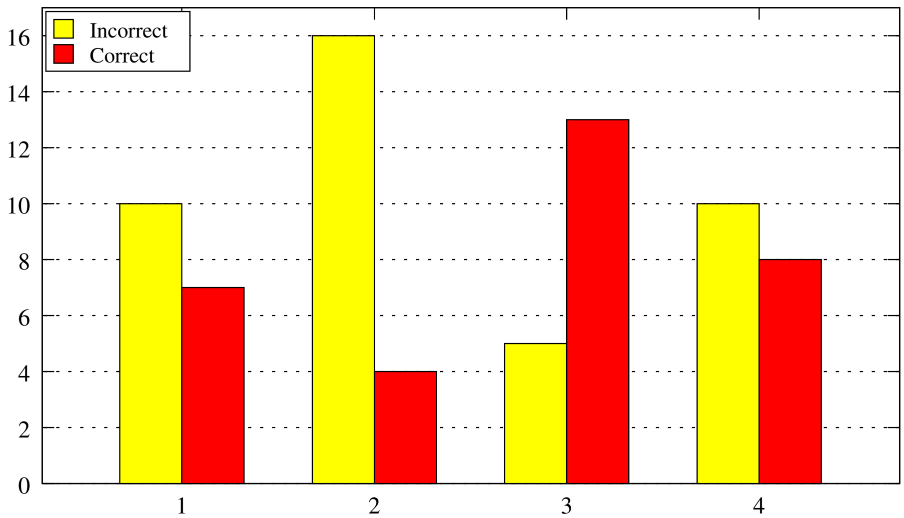
4. *Given a rectangle with length ℓ and width w, you can roll it into a tube in two ways. Write the ratio of the volume of the tube with height ℓ to the volume of the tube with height w.*

This is a pretty simple straightforward algebra problem that doesn't require much manipulation. 47 students got it, although many didn't simplify the answer, and 57 students missed it.

For my 73 students, 41 missed it and 32 got it right. This should be a much more interesting problem to look at than when nearly everyone gets a problem right (or wrong).

In fact, it seems that the results don't affect the top and bottom quartiles, but has a significant connection to what happens in the second and third quartiles.

Figure 4: Number of students



4. TRIGONOMETRY

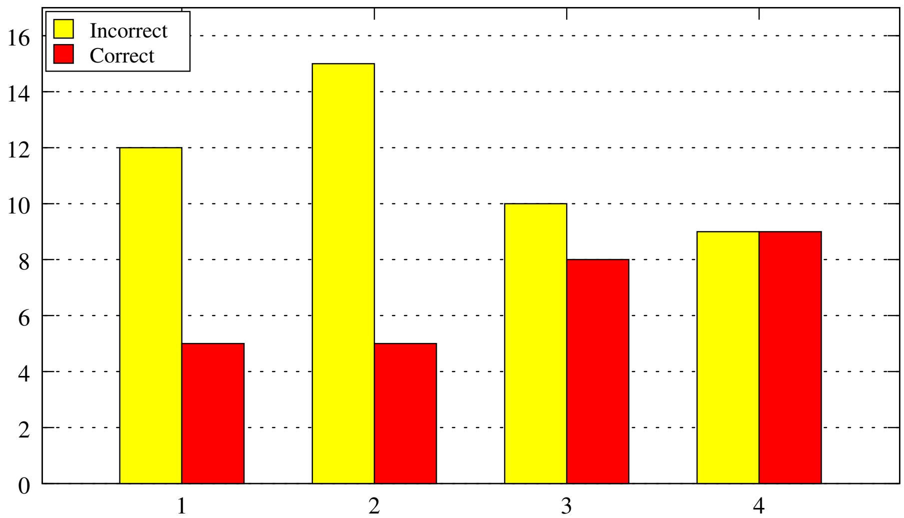
5. *Let* $0 = -4\sin(\theta) + 2\cos(\theta)$. *Solve for* θ.

The reason for this problem was because it came up early in passing on an exam a couple of years ago and much depended on it. I had assumed that it wouldn't be a problem but found many of my students had a very superficial grasp of trig. So, I wanted to quantify this and I reduced the big problem this came up in to just this simple equation. 31 students got the correct answer, of which there are many acceptable versions. 69 missed it entirely. Another 4 came close with $\tan(\theta) = 1/2$ but didn't go the extra step and solve for θ to get $\arctan(1/2) = \theta$.

In the case of my 73 students, 46 of them missed it, 3 didn't quite finish it (but I will count them as correct here), and 24 got it right.

It looks pretty clear to me that being able to do really simple trig problems is a real advantage in Calculus.

Figure 5: Number of students



5. MY MISTAKE

6. *Find the coordinates for the minimum value of the function*

$$y = x^2 - 3x + 4.$$

DO NOT USE CALCULUS.

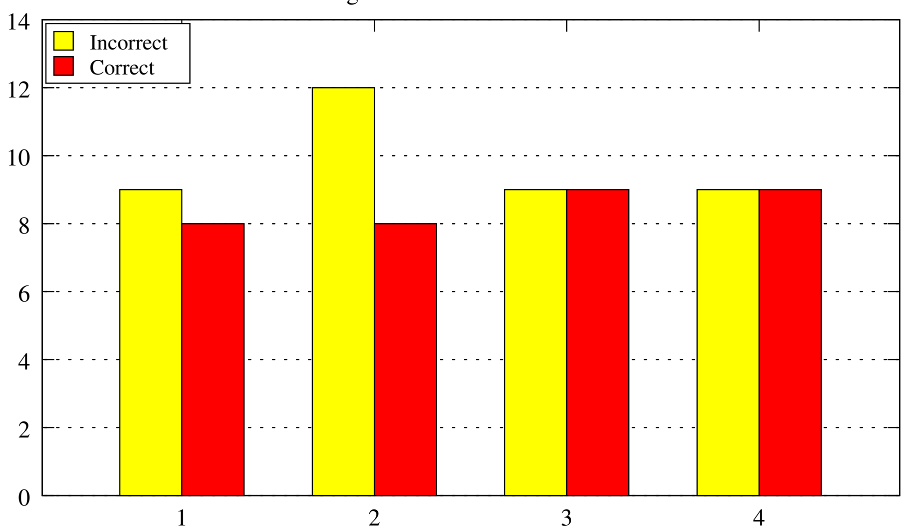
This is pretty simple using calculus, and it was my intent that students should have to complete the square to do this. I obviously wasn't thinking because there are many ways to get this much easier than completing the square. Many students plotted a few points and figured that the minimum was between two equal values by symmetry. Some actually could see that because $x^2 - 3x = x(x-3)$, the line of symmetry had to be $x = 3/2$, which was real math but didn't require any computation, so it was often hard to tell exactly what anyone did. Some remembered the formula for the value of x for the line of symmetry, for example.

Although it wasn't what I wanted, the results are still interesting. 2 students actually did complete the square to solve the problem. They should probably be docked for not seeing easier ways to do it. 10 cheated and used calculus. 16 obviously remembered the formula. 26 got the right answer, probably using some symmetry analysis or just by guessing. It was hard to tell. What I could tell though was that 50 students just plain missed it.

For my 73 students, I'm inclined to say that someone who used calculus got it wrong. Notice it rather explicitly says not to use calculus?

Counted this way, 39 got it wrong and the other 34 got it right somehow. This turns out to be pretty unexciting and uninteresting.

Figure 6: Number of students



6. Some real calculus

7. *We want to make a cylindrical can of radius r and height h with a given volume V. It has a top and bottom as well as the cylindrical tube. What is the ratio, h/r, for the can with the minimum surface area for this volume? (You can use calculus.)*

This is a good solid differential calculus question. My course is Calculus II though, and if there is anything you would want students to know about differential calculus after taking a Calculus I course, it would be to be able to solve problems like this. Although not the easiest problem, this is still a basic multi-step max/min calculus problem. This problem should be considered a prerequisite for Calculus II. Unfortunately, only 2 students solved it. 102 missed it.

Only 1 of those 2 students took and finished my course and that student had the third highest score on the final exam (out of 88 students).

Not enough information to make a worthwhile graph here.

7. Enter (high school in) China

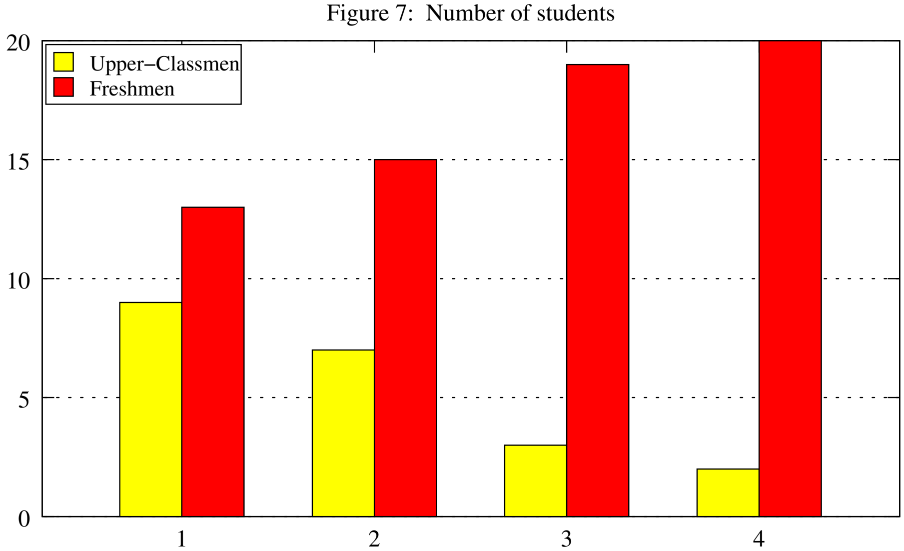
8. *This problem is just here to give you something to do for the rest of the class. Make sure you do the other problems first. This is a* *simplified version of a problem taken off a Chinese college admissions test. Consider the parabola that goes through the points (-4,0), (0,-4), and (2,0). Find the maximum area of a triangle made by the points (-4,0), (0,-4) and a point on the parabola in the third quadrant.*

I was wrong when I wrote this. This was just one part of a problem from a **high school** admissions test in China, not a college admissions test. As a consequence of being wrong, I solved it using calculus. When I realized that Calculus wasn't allowed, probably even on a college admissions test, I did go back and solve it using algebra. None of my 104 students solved this problem, so likewise, my subset of 73 students didn't either. I could graph this, but it isn't a good idea.

8. Miscellaneous

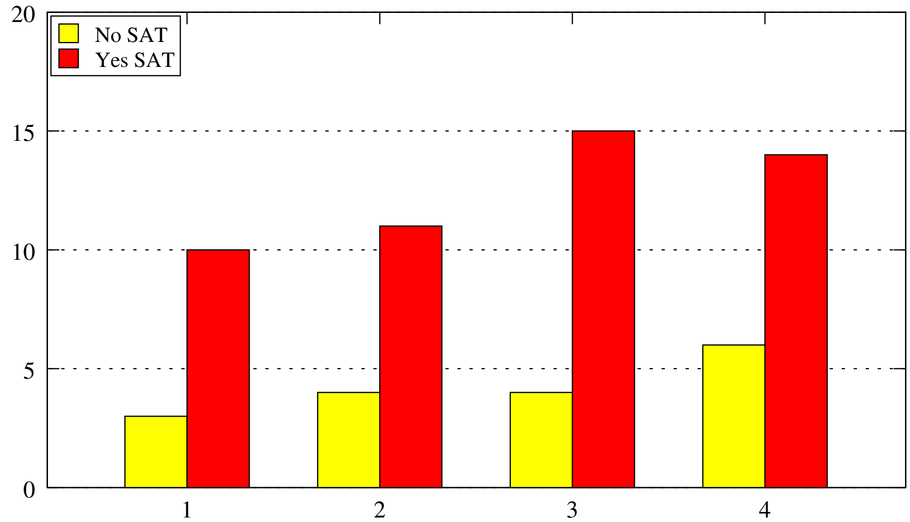
On the whole, we suspect weaker students put off a required calculus course or, at least, if you put it off, you are rusty. The data support something along these lines. For this we go to the whole class of 88 and not worry about the little test I gave. There were 21 upper class students and 67 freshmen.

Figure 7: Number of students



Of the 67 Freshmen in the course, only 50 of them had SAT math scores, so I was a bit curious about how those without compare with those who submitted. I wondered if they had a different admission standard, but it seems not.

Figure 8: Number of students



Next, I was interest in whether or not the SAT math score was a good predictor. I've broken up the SAT scores two ways for the following graphs, and if you can make anything of them, let me know. I should point out that although my range goes down to 590, only one student had a 590 and the next lowest was 640, where, again, there was only one student (they both did poorly, i.e. ended in the first quartile of the final exam). The range really starts at 660.

Figure 8: Number of students

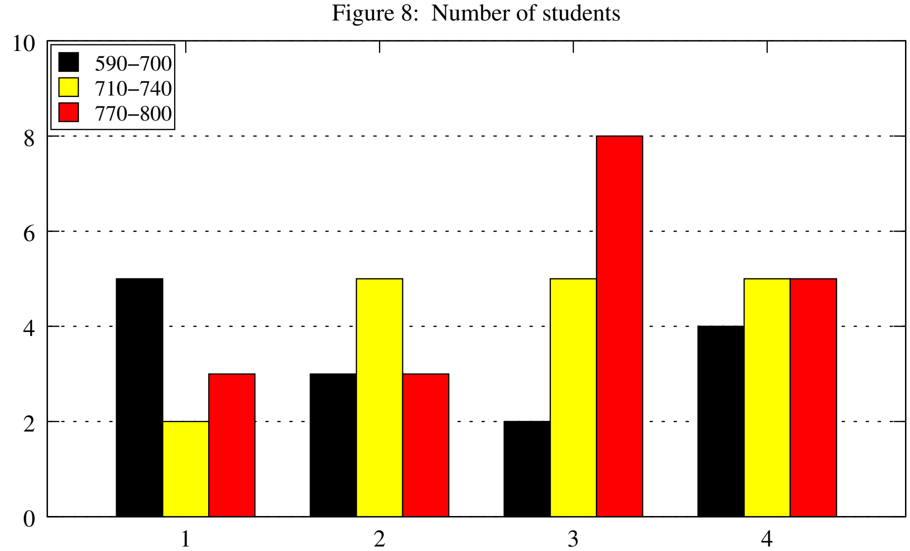
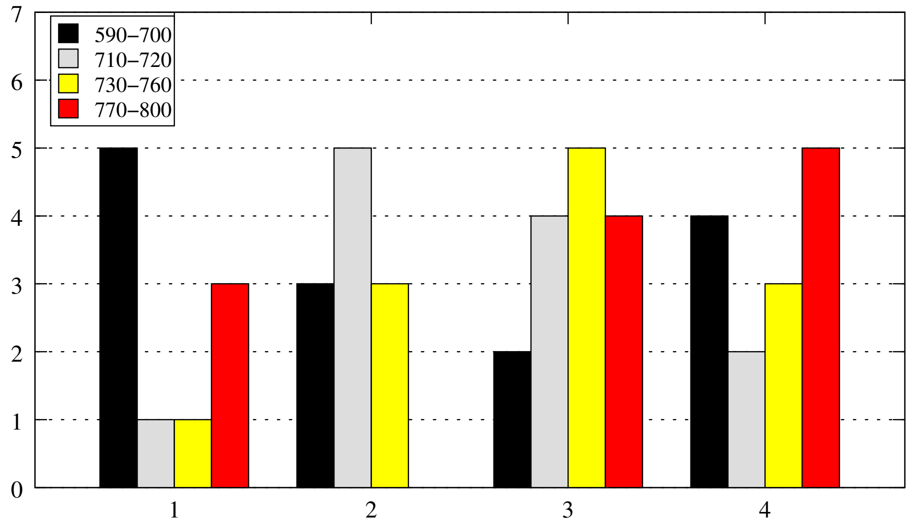
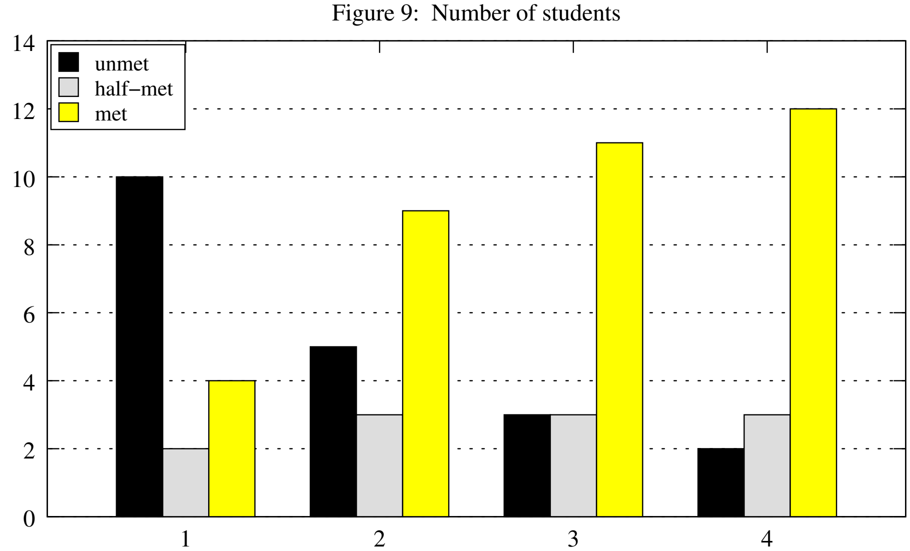


Figure 8: Number of students

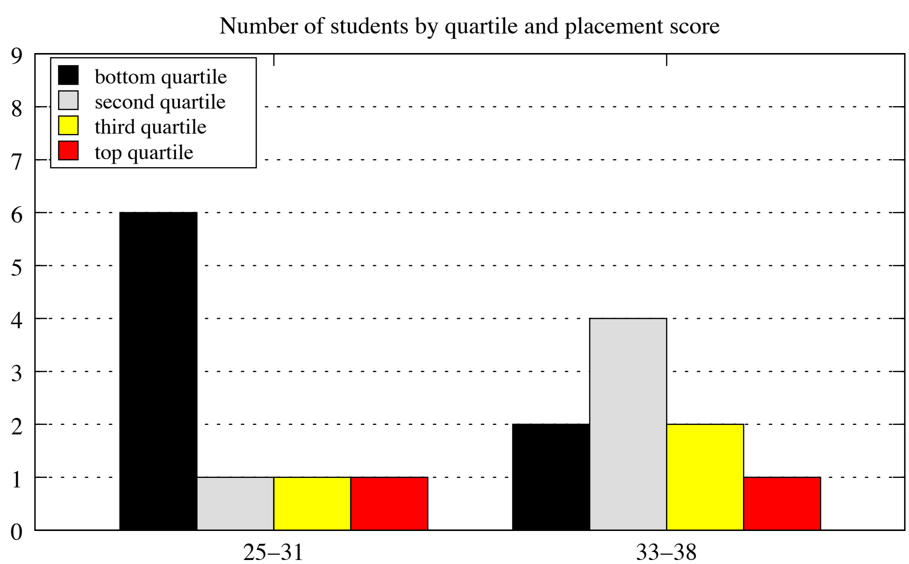


Finally, now that we are the end, we get something interesting. Of the 67 freshmen in the class, 20 had blatantly not met the requirement of having a 5 on the Advance Placement Calculus AB exam. Another 11 had met the requirement of having a 3 or 4 on the BC exam, but their sub-score for the AB part was not a 5. The other 36 had a 5 on the AB or a 5 on the AB sub-score of the BC exam. We call these unmet, half-met and met in the graph.

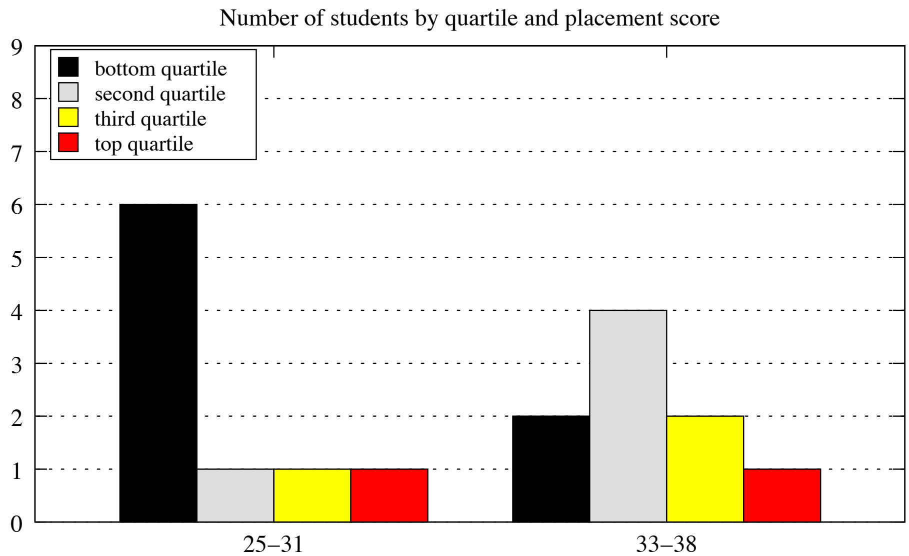
Figure 9: Number of students



Well, we actually give a placement test for those who want in but didn't have the appropriate AP Calculus score. Of these 20 students who did not meet the formal Advance Placement test requirement to get into the course should have done so by way of our placement test. If they placement test is good, they should do as well as the other students but the results above show they didn't. 2 of these 20 students seem not to have taken the placement test at all and they end up in the bottom quartile on the final exam for the course. That only leaves 18 students to "analyze." The placement test scores range from 25 to 38. No one got a 32 and there are 9 below 32 and 9 above. Splitting things up in this way, let's first look at the quartiles students end up in based on their placement test being high or low. Certainly if the cut off was at 32, things would improve, see graph.



However, I don't think the distribution for those above 32 is all that great. The two students with score of 33 both ended up in the first quartile and the student with a 34 is in the second. If we make our cutoff at 35 and above, we get a new graph.



This is probably acceptable, i.e. a cutoff of 35 or above to get into Calculus II. There are two students in the top half with scores less than 35, but they are overwhelmed by the 10 students in the bottom half. At least with a cutoff of 35, half are in the top half and half in the bottom half. Still, only 6, so pretty small numbers. The point remains that the placement exam isn't working as is. The cutoff should be higher, but I actually don't know what the cutoff is at present.

9. SUMMARY

This little test of mine didn't have the predictive value I had hoped it would have. There seemed to be a bit that implied being good at arithmetic was helpful, but not much. A potential problem here is that these are elite students with high SAT math scores. I suspect that the arithmetic would be a better predictor in an environment where the average SAT score was much lower. No way for me to check that.

On the other hand, the simple algebra problem gave pretty interesting results and so did the trig question, although somewhat different results.

I'm a little surprised that the basic algebra max/min problem didn't yield better results, but so be it.

On the other hand, the last two questions gave dismal results that don't give us any way to interpret them. Not being able to solve a basic calculus problem or a Chinese high school admissions problem, well, it is just sad.

The main lesson from all of this is the importance of meeting some substantive prerequisite that shows a student is ready for Calculus II. The AP AB 5 did okay, but our placement test, or at least the cutoff score, needs tweaked.

This last is very important. Students who are placed in a course they are not properly prepared for are not happy and they don't do well. They also alter the very nature of the course in a negative way. Students who are prepared are better off not being in a classroom with unprepared students, and, needless to say, this also works better for the professor.