Spring 2002 Algebra Qualifying Exam

March 10, 2002

1. How many elements of order 7 are there in a simple group of order 168?

2. Let m be an integer ≥ 2 and $\mathbf{Z}[X]$ be the polynomial ring over $\mathbf{Z}$. Find a condition on m so that the ideal (m, X) in the ring is maximal.

3. Prove that the group of automorphisms of the field $\mathbf{R}$ of real numbers is trivial.

4. For a field K, let $SL_2(K)$ be the special linear group over K, i.e. the group of $2 \times 2-$ matrices over K with determinant 1, and let $PSL_2(K)$ be the quotient of $SL_2(K)$ by its center, i.e. the projective special linear group. Find the order of $PSL_2(\mathbf{F}_7)$ where $\mathbf{F}_7$ denotes the finite field of 7 elements.

5. Let $\zeta = e^{\frac{2\pi i}{5}}$ and $K = \mathbf{Q}(\zeta)$ the field generated by ζ over the field of rational numbers. Prove that K contains $\sqrt{5}$.