1. Show that every group of order $p^2$, $p$ a prime, is Abelian. Show that up to isomorphism there are only two such groups.

2. Let $K$ be a field. A polynomial $f(x) \in K[x]$ is called separable if, in any field extension, it has distinct roots. Prove that:
   (a) if $K$ has characteristic 0, then each irreducible polynomial in $K[x]$ is separable; and
   (b) if $K$ has characteristic $p \neq 0$, then an irreducible polynomial $f(x) \in K[x]$ is separable if and only if has no form $g(x^p)$ where $g(x) \in K[x]$.
   Give an example of an inseparable irreducible polynomial.

3. Prove that if a linear operator on a complex vector space is diagonal in some basis, then its restriction on any invariant subspace $L$ is also diagonal in some basis of $L$.

4. A differentiation of a ring $R$ is a mapping $D : R \to R$ such that, for all $x, y \in R$,
   (1) $D(x + y) = D(x) + D(y)$; and
   (2) $D(xy) = D(x)y + xD(y)$.
If $K$ is a field and $R$ is a $K$-algebra, then its differentiation are supposed to be over $K$, that is,
   (3) $D(x) = 0$ for any $x \in K$.
   Let $D$ be a differentiation of the $K$-algebra $M_n(K)$ of $n \times n$-matrices. Find a matrix $A \in M_n(K)$ such that $D(X) = AX -XA$ for all $X \in M_n(K)$.

5. Prove the existence of a 1-dimensional invariant subspace for any 5-dimensional representation of the group $A_4$ (the alternating group of degree 4).