Algebra Qualifying Exam Spring 2010

May 13, 2010 (150 minutes)

Do all problems. All problems are equally weighted. Show all work.

1. Let G be a non-abelian group of order p^3, here p is prime. Determine the number of distinct conjugacy classes in G.

2. Let R be a ring such that $r^3 = r$ for all $r \in R$. Show that R is commutative. (Hint: First show that r^2 is central for all $r \in R$.)

3. Compute Galois groups of the following polynomials.

(a). $x^3 + t^2x - t^3$ over k, where $k = \mathbb{C}(t)$ is the field of rational functions in one variable over complex numbers $\mathbb{C}$.

(b). $x^4 - 14x^2 + 9$ over $\mathbb{Q}$.

4. Let V be a n-dimensional vector space over a field k. Let $T \in End_k(V)$.

(a). Show that $tr(T \otimes T \otimes T) = (tr(T))^3$. Here $tr(T)$ is the trace of T.

(b). Find a similar formula for the determinant $det(T \otimes T \otimes T)$.

5. Classify all non-commutative semi-simple rings with 512 elements. (You can use the fact that finite division rings are fields.)

6. Let G be a group with 24 elements. **Use representation theory** to show that $G \neq [G, G]$. (Here $[G, G]$ is the commutator subgroup of G.)