Algebra Qualifying Exam, Spring, 2014

May 14th, 9:00-12:00

1. Find the number of colouring of faces of a cube in 3 colours. Two colourings are equal if they are the same after a rotation of the cube. [Hint Use the Burnside formula

   $$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|,$$

   where a group G acts on a set X, X/G is the set of orbits, and, for every $g \in G$, X^g is the fixed subset of g in X.]

2. Proof that all groups of order < 60 are solvable.

3. Let L/K be a Galois extension of degree p with $\mathrm{char} K = p$. Show that $L = K(\theta)$, where θ is a root of $x^p - x - a, a \in K$, and, conversely, any such extension is Galois of degree 1 or p.

4. Proof that a finite dimensional associative algebra over a field is a division algebra if and only if it has no zero divisors.

5. Find the table of characters for S_4.