Algebra Qualifying Exam, Spring, 2016

May 11th, 12:30-3:30

All six problems are equally weighted. (The problem parts need not be equally weighted.) Explain clearly how you arrive at your solutions, or you risk losing credit.

1. Classify all groups of order 66, up to isomorphism.

2. Let $F \subset K$ be an algebraic extension of fields. Let $F \subset R \subset K$ where R is a F-subspace of K with the property such that $\forall a \in R$, $a^k \in R$ for all $k \geq 2$.

   (1). Assume that $char(F) \neq 2$. Show that R is a subfield of K.

   (2). Give an example such that R may not be a field if $char(F) = 2$.

3. Determine the Galois group of $x^6 - 10x^3 + 1$ over $\mathbb{Q}$.

4. Let V and W be two finite dimensional vector spaces over a field K. Show that for any $q > 0$,

$$\bigwedge^{q}(V \oplus W) \cong \sum_{i=0}^{q} (\bigwedge^{i}(V) \otimes_K \bigwedge^{q-i}(W)).$$

5. Prove that a finite dimensional algebra over a field is a division algebra if and only if it does not have zero divisors.

6. Let A be a semi-simple finite dimensional algebra over $\mathbb{C}$, and let V be a direct sum of two isomorphic simple A-modules. Find the automorphism group of the A-module V.