1. Prove the following statement without using Ergoroff’s Theorem: Suppose \( \{f_k\}_{k=1}^{\infty} \) is a sequence of measurable functions defined on a measurable set \( E \), \( f_k \to f \) a.e. on \( E \) and there exists \( g \in L^1(E) \) such that \( |f_k| \leq g \) for all \( k \). Given \( \epsilon > 0 \), there exists a closed set \( A_\epsilon \) such that \( m(E \setminus A_\epsilon) < \epsilon \) and \( f_k \to f \) uniformly on \( A_\epsilon \).

2. Let \( f \in L^1(\mathbb{R}) \) and define \( E_\alpha = \{ x : |f(x)| > \alpha \} \). Prove that
   \[
   \int_{\mathbb{R}} |f(x)| \, dx = \int_{0}^{\infty} m(E_\alpha) \, d\alpha.
   \]

3. Let \( f : \mathbb{R} \to \mathbb{R} \) be a measurable function. Prove the following statement: There exists \( M > 0 \) such that \( |f(x) - f(y)| \leq M|x - y| \) for all \( x, y \in \mathbb{R} \) if and only if \( f \) is absolutely continuous and \( |f'| \leq M \).

4. (a) Prove that the operator \( T : L^2([0,1]) \to L^2([0,1]) \) defined by setting \( T[f](x) = xf(x) \) is continuous and symmetric (self-adjoint).
   
   (b) Prove that \( T \) is not compact.

5. Let \( D = \{ z \in \mathbb{C} : |z| < 1 \} \) and \( f : D \to D \) be a holomorphic function. Prove
   \[
   \frac{|f(0)| - |z|}{1 + |f(0)||z|} \leq |f(z)| \leq \frac{|f(0)| + |z|}{1 - |f(0)||z|}, \quad \forall z \in D.
   \]

6. For \( t \in \mathbb{R} \), compute
   \[
   \lim_{A \to \infty} \int_{-A}^{A} \frac{\sin x}{x} e^{i\pi t} \, dx.
   \]

7. Let \( U \subset \mathbb{C} \) be an open set, \( f : U \to \mathbb{C} \) be a holomorphic function and \( z_0 \in U \). Prove that if \( f'(z_0) = 0 \), then \( f \) is not one-to-one in any neighborhood of \( z_0 \).

8. Prove that if \( f \) is an entire function and \( |f(z)| \leq a + b|z|^k \) for all \( z \in \mathbb{C} \) where \( a, b \) and \( k \) are positive real numbers, then \( f \) is a polynomial.