ALGEBRA EXAMTOPICS

Linear Algebra

Abstract vector spaces; subspaces; dimension; inner products and orthogonal bases; matrices and linear transformations; matrix algebras and groups; determinants and traces; eigenvectors and eigenvalues, canonical forms; reduction of quadratic forms; bilinear forms; dual spaces; tensor products and tensor algebras.

Group Theory

Groups and homomorphisms, Sylow, Jordan-Holder and Krull-Schmidt theorem; representations of finite groups; permutation representations; nilpotent, solvable and simple groups; matrix groups; free groups; extensions and group cohomology.

Commutative Rings

Factorization and localization, chain conditions, Hilbert basis theorem, integral extensions, primary decompositions, Hilbert Nullstellensatz.

Module Theory

Free and projective modules; tensor products; irreducible modules and Schur's lemma; semisimple, simple and primitive rings; density and Wederburn theorems; the structure of finitely generated modules over principal ideal domains, with application to abelian groups and canonical forms; catagories and functors; complexes, cohomology; Tor and Ext.

Field Theory

Field extensions, algebraic extensions, transcendence bases; cyclic and cyclotomic extensions; solvability of polynomial equations; finite fields; separable and inseparable extensions; Galois theory.

References

Hungerford, *Algebra*

Lang, *Algebra,* (2^{nd} edition)

Jacobson, *Basic Algebra I*

Rotman, *The theory of Groups*