REAL AND COMPLEX ANALYSIS EXAM

TOPICS

I. Real Analysis

1. Knowledge of material from undergraduate analysis: topics such as open and closed sets, compactness (incl. Heine-Borel theorem), continuity (incl. uniform continuity), uniform convergence and the Arzela-Ascoli theorem.

2. Measure theory: Measurable sets and functions, outer measure, construction of Lebesgue measure. Lusin's theorem. Notions of convergence involving measure (pointwise a.e., convergence in measure). Egorov's theorem.

3. Lebesgue integral: Definition of $\int_X f(x)d\mu(x)$ and relation to the distribution function $\mu\{x : f(x) > t\}$. Fatou's Lemma and dominated convergence. $L^1(X, d\mu)$ as a normed space. Relation of convergence in L^1 to pointwise a.e. convergence and convergence in measure. Product measure and the Fubini theorem. Relation between Lebesgue and Riemann integrals. Riesz representation theorem concerning positive linear functionals on $C(X)$.

4. L^p spaces: Jensen, Holder and Minkowski inequalities. Completeness. Duality of L^p and L^q for $\frac{1}{p} + \frac{1}{q} = 1$. Bounded functionals, weak convergence on L^p. Uniform boundedness principle for $L^p(X, \mu)$. Approximation of L^p functions on $\mathbb{R}^n$ by smooth functions. Weak compactness of unit ball in L^p. $L^2(X, d\mu)$. Hilbert space: Bessel inequality, orthonormal bases.

5. Fourier analysis: Fourier transform $\mathcal{F}$ on $L^1(\mathbb{R}^n)$ and $L^1(S^1)$ (S^1 = unit circle). Riemann-Lebesgue lemma. Plancherel theorem ($\mathcal{F}$ as a unitary operator on $L^2(\mathbb{R}^n)$). Parseval inequality for $\mathcal{F}$ on S^1. Inversion formula. Convolution. Hausdorff-Young inequality.

References:

W. Rudin, Real and Complex Analysis, McGraw Hill.
E. Lieb and M. Loss, Analysis, AMS Graduate Studies in Math. 14.
G. B. Folland, Real Analysis, Wiley Interscience.

II. Complex Analysis

1. Analytic (= holomorphic) functions: Power series, radius of convergence, Cauchy-Riemann equations. Liouville theorem, Cauchy estimates. Uniform limits. Discreteness of zeros.

2. Meromorphic functions: Riemanns removable singularities theorem, poles, Laurent series, Cauchy integral formula, residue theorem, residue calculus.

3. Local behaviour of holomorphic functions: Argument principle, zeros of holomorphic functions, Rouches theorem, Hurwitzs theorem. Maximum modulus principle. Open mapping theorem.

4. Holomorphic mappings: Riemann sphere, linear fractional transformations, conformal mapping, Schwarz lemma, normal families, Riemann mapping theorem.

5. [**This topic will not be included in the September 2005 exam.**] Harmonic functions (on the plane): Poisson integral formula, subharmonic functions, Dirichlet problem.

References:

R. E. Greene and S. G. Krantz, Function Theory of One Complex Variable
J. B. Conway, Functions of One Complex Variable
L. Ahlfors, Complex Analysis