COMPLEX ANALYSIS CORE QUALIFYING EXAM, SPRING 2001

Directions: Do **FIVE** of the following six questions; they are weighted equally. Label clearly which five that you want graded (otherwise only first five will be). Show your work.

Question 1. Suppose that f, g are entire holomorphic functions with $|f(z)| \leq |g(z)|$ for all $z \in \mathbf{C}$. Prove that there is a constant $c \in \mathbf{C}$ so that $f = c\,g$.

Question 2. Find the number of zeros of the function $f(z) = 2z^5 + 8z - 1$ in the annulus $1 < |z| < 2$.

Question 3. Assume that f_n is holomorphic in $|z| < 1$ and $|f_n| \leq 10$. Assume also that $\lim_{n\to\infty} f_n(2^{-j})$ exists for each $j = 1, 2, \ldots$. Prove that $\lim_{n\to\infty} f_n(z)$ exists for all z with $|z| < 1$.

Question 4. Let $u(z) > 0$ be a positive harmonic function in the punctured plane $0 < |z|$. Show that u must be constant.

Question 5. Let f be a non-constant holomorphic function in the annulus $1 < |z| < 2$ with $|f| \equiv 5$ on the boundary. Show that f has at least two zeros.

Question 6. Let $P(z)$ be a polynomial. Show that all zeros of $P'(z)$ lie in the convex hull of the zeros of $P(z)$.