Local zeta functions and the arithmetic of moduli spaces
March 22–26, Johns Hopkins University

Titles and Abstracts

Hiroki Aoki
On the structure theorem for modular forms ... Igusa’s result and beyond
In this talk we treat the structure theorem of the graded ring of modular forms of several variables. Determining the structure is not easy in general. The first result was given by Professor Jun-Ichi Igusa in 1962. This was on the graded ring of Siegel modular forms of degree 2 of even weights. And then in 1964, he determined the graded ring of Siegel modular forms of degree 2 of integral weights. After these results, we have many improvements in the methods of the determination. Here we study Igusa’s result and some improvements after that.

Ana Caraiani
On torsion in the cohomology of certain unitary Shimura varieties
In this talk, I will discuss joint work in progress with Peter Scholze on torsion in the cohomology of certain unitary Shimura varieties. I will describe a way of computing this cohomology using perfectoid Shimura varieties, the Hodge–Tate period morphism, and Igusa varieties. I will also mention joint work which applies these results to elliptic curves over imaginary quadratic fields.

Ivan Cheltsov
Igusa quartic and and Wiman–Edge sextics
We describe \( \mathbb{Q} \)-factorization of the double cover of the four-dimensional projective space branched over the Igusa quartic. As an application, we give an alternative proof of Beauville’s result that every quartic threefold defined by

\[
\sum_{i=0}^{5} x_i = \sum_{i=0}^{5} x_i^4 - t \left( \sum_{i=0}^{5} x_i^2 \right)^2 = 0
\]

in \( \mathbb{P}^5 \) is irrational provided that \( t \neq \frac{1}{6}, \frac{1}{10}, \frac{1}{2}, \frac{7}{10} \). Namely, we show that these threefolds are birational to conic bundles over quintic del Pezzo surfaces whose degeneration curves are contained in the pencil studied by Wiman and Edge. This is a joint work with Alexander Kuznetsov and Constantin Shramov (Moscow).
Raf Cluckers

*Uniform $p$-adic integration and applications*

As a concrete variant of motivic integration, we will discuss uniform $p$-adic integration and constructive aspects of results involved. Uniformity is in the $p$-adic fields, and, for large primes $p$, in the fields $\mathbb{F}_p(t)$. Using real-valued Haar measures on such fields, one can study integrals, Fourier transforms, etc. We follow a line of research that Jan Denef started in the eighties, with in particular the use of model theory to study various questions related to $p$-adic integration. A form of uniform $p$-adic quantifier elimination is used. Using the notion of definable functions, one builds constructively a class of complex-valued functions which one can integrate (with respect to some of the variables) without leaving the class. One can also take Fourier transforms in the class. Recent applications in the Langlands program are based on Transfer Principles for uniform $p$-adic integrals, which allow one to get results for $\mathbb{F}_p((t))$ from results for $\mathbb{Q}_p$, once $p$ is large, and vice versa. These Transfer Principles are obtained via the study of general kinds of loci, some of them being zero loci. More recently, these loci are playing a role in the uniform study of $p$-adic wave front sets for (uniformly definable) $p$-adic distributions, a tool often used in real analysis. This talk contains various joint work with Gordon, Hales, Halupczok, Loeser, Raibaut.

Jan Denef

*Igusa fibre integrals over local fields*

Examples of Igusa fibre integrals are the local singular series from analytic number theory. Igusa obtained fundamental results about these in the univariate case and asked how this could generalize to the multivariate case. First results were obtained by Loeser (1988), Lichtin (1997), and myself (1998). Later Cluckers and Loeser (2005) proved, using cell decomposition, that these multivariate fibre integrals (and generalizations) are specializations of motivic constructible functions. They deduced from this their transfer principle for integrals over local fields of large residue characteristic.

I will explain how to obtain a slightly stronger result in the special case that there are no additive characters involved, by using a theorem of Abramovich and Karu on flat toroidalization of morphisms.

Carel Faber

*On the cohomology of the moduli spaces of $n$-pointed curves of genus at most three*

Consider the case of stable pointed curves for simplicity. The answers for genus 0 and 1 have been known for a long time. The answer for genus 2 follows from Petersen’s proof of a conjecture of Van der Geer and the author. In genus three, the answer for $n$ at most 14 can be deduced from a recent result of Chenevier and
Lannes, if one accepts the Langlands conjecture. The recent work of Mégarbané allows us to identify nearly all the cohomology for \( n \) at most 19. Joint work with Bergström, Cléry, and Van der Geer.

**Julia Gordon**

*Measures, orbital integrals, and product formulas for isogeny classes of abelian varieties*

In 2003, E.-U. Gekeler gave a formula for the number of elliptic curves in an isogeny class (over a finite field), based on probabilistic and equidistribution considerations. This formula is in a sense similar to Siegel’s formula expressing the size of a genus of a quadratic form as a product of local densities. It is well-known that Siegel’s formula is equivalent to the computation of the Tamagawa volume of the orthogonal group. In the same spirit, there is a formula, due to Langlands and Kottwitz, expressing the cardinality of an isogeny class of principally polarized abelian varieties as an orbital integral. It turns out that by carefully comparing different natural measures on the orbits, one can see a direct connection between Gekeler’s formula and the formula of Langlands and Kottwitz, and therefore generalize Gekeler-style formula to higher dimension. In the process we encounter some questions that appear classical but seem to be surprisingly difficult to answer, which I am hoping to discuss. This is a joint project with Jeffrey Achter, Salim Ali Altug, and Luis Garcia.

**Richard Hain**

*Introduction to Mixed Modular Motives*

In this talk I will give an introduction to Francis Brown’s theory of “mixed modular motives” and their periods, which include “multiple modular values”. Multiple modular values form a tannakian category whose semi-simple objects are the motives of Hecke eigenforms. Their periods include iterated integrals of modular forms, which have been studied by Manin and Brown. I will motivate the definition of mixed modular motives by giving an overview of the theory of mixed Tate motives, unramified over the integers, and their periods, multiple zeta values, which occur as periods of the unipotent fundamental group of the projective line minus 3 points.

**Thomas Hales**

*The Spherical Hecke algebra, partition functions, and motivic integration*

This talk will describe a proof of the Langlands–Shelstad fundamental lemma for the spherical Hecke algebra for every unramified \( p \)-adic reductive group \( G \) in large positive characteristic. The proof is based on the transfer principle for constructible motivic integration. To carry this out, we introduce a general family of partition functions attached to the complex \( L \)-group of the unramified \( p \)-adic group \( G \). Our partition functions specialize to Kostant’s \( q \)-partition function
Tomoyoshi Ibukiyama

*Universal automorphic differential operators on Siegel modular forms*

Holomorphic differential operators acting on Siegel modular forms and preserving the automorphy when restricted to a fixed smaller domain are important objects. They are applied to obtain critical values of $L$-functions, to construct modular forms (e.g. Igusa’s $\chi_{35}$), or to lift modular forms, and besides, they are also interesting as a theory of special functions whose prototypes are the Gegenbauer polynomials. We will explicitly give a universal object defined by a certain concrete series such that all such differential operators as above are obtained from it, in case of the restriction to any fixed diagonal blocks of the Siegel upper half space of any degree.

Kiyoshi Igusa

*Polylogarithms and equivariant higher Franz–Reidemeiser torsion*

This is a report on ongoing joint work with Tom Goodwillie based on earlier joint work with Goodwillie and Ohrt. In this project we generalize a very elegant topological construction of Allen Hatcher to explicitly construct exotic smooth structures on smooth manifold bundles and compute their higher Franz–Reidemeister torsion invariants.

The talk will give a brief description of the topological problem and the solution, then concentrate on the known and unknown properties of polylogarithms of roots of unity which are used or circumvented in the proofs.

Kazuya Kato

*Height functions for motives*

We define various height functions for motives over number fields. Many interesting problem arise concerning them. Using them, we can formulate motive versions of Vojta conjectures and Manin conjectures which were originally formulated using height functions for rational points of algebraic varieties over number fields.
Nick Katz

Finite monodromy groups of rigid local systems on $\mathbb{A}^1$

We formulate some conjectures about the precise determination of the monodromy groups of certain rigid local systems on $\mathbb{A}^1$ whose monodromy groups are known, by results of Kubert, to be finite. We prove some of them.

François Loeser

Motivic curve counting

A well-studied question put forward by Manin is that of an asymptotic expansion for the number of rational/integral points of bounded height. A basic tool in that study is the height zeta function which is a Dirichlet series. Around 2000, Peyre suggested to consider the analogous problem over function fields, which has then an even more geometric flavor since it translates as a problem of enumerative geometry, namely counting algebraic curves of given degree and establishing properties of the corresponding generating series.

In this talk I will present joint work with Antoine Chambert-Loir on a geometric version of a result of Chambert-Loir and Tschinkel on integral points of bounded height for equivariant compactifications of additive groups. Key use is made of motivic integrals of Igusa type and of a motivic Poisson formula due to Hrushovski and Kazhdan. We shall end the talk with some recent results by Margaret Bilu.

Takayuki Oda

How to get non-trivial cohomology classes in Shimura varieties, a case study

For holomorphic automorphic forms on arithmetic quotients of Hermitian symmetric domains, we know some methods to get some “explicit” construction: theta series, and Eisenstein series. But we still know little for non-holomorphic harmonic forms.

In my talk, (a) first I review fundamental results on cohomological representations, starting from the Matsushima isomorphism; (b) second we review old results on Green currents for Shimura subvarieties embedded in a Shimura variety in the canonical way. This is a very natural way to generalize the Kronecker limit formula, which is also considered as a “generalization” of Eisenstein series; (c) we review some technically difficult problems on cohomological representations, which prevent further generalizations.

Dan Petersen

Tautological classes with twisted coefficients

Let $M_g$, for $g \geq 2$, be the moduli space of smooth curves of genus $g$. Mumford defined a subring $R(M_g)$ of the Chow ring $\text{CH}(M_g)$ called the tautological ring. I explain how to associate to any irreducible algebraic representation of $\text{Sp}(2g)$
a relative Chow motive $V_{\lambda}$ over $M_g$, and how to define a tautological subgroup $R(M_g, V_{\lambda})$ inside $\text{CH}(M_g, V_{\lambda})$. Computing $R(M_g, V_{\lambda})$ for all $\lambda$ is equivalent to computing the tautological rings of all fibered powers of the universal curve over $M_g$ simultaneously. We are able to completely determine $R(M_g, V_{\lambda})$ for all $\lambda$ when $g$ is at most 4. A particular consequence is that the tautological rings of all fibered powers of the universal curve over $M_g$ satisfy Poincaré duality in these genera. This was previously known only in genus 2. We also obtain results about conjectural failures of Poincaré duality for $g \geq 5$; specifically, we can show that if certain modified diagonal cycles on products of very general curves are nonzero in Chow, then Poincaré duality fails in the tautological ring. (Joint with Mehdi Tavakol and Qizheng Yin.)

Takeshi Saito

Characteristic cycle of an $\ell$-adic sheaf

For an $\ell$-adic sheaf on a smooth variety over a perfect field, its characteristic cycle is defined as a $\mathbb{Z}$-linear combination of irreducible components of the singular support, defined by Beilinson as a closed conical subset of the cotangent bundle. It gives an analogue of that defined by Kashiwara–Schapira in a transcendental setting. We discuss its properties, including a recent progress on the compatibility with proper direct image.

Riccardo Salvati Manni

Theta-constants: the influence of Igusa’s work

We will try to illustrate fundamental results on theta-constants obtained by Igusa over a period of about 20 years. We will show how his work is strongly influencing research on the subject.

Tetsuji Shioda

Mordell–Weil Lattice and Invariant Theory

We discuss the Mordell–Weil Lattice of rational elliptic surfaces and the invariant theory of the Weyl group $W(E_8)$, and applications to arithmetic and geometry. A variant for $W(E_6)$ has a nice application to the classical topic of 27 lines on a cubic surface.

Takahiro Shiota

Soliton equations and the Schottky problem

We discuss the Burchnall–Chaundy–Krichever theory and its applications to solving Novikov’s and Welters’ conjectures on the Schottky problem.
Yuri Tschinkel

*Igusa Integrals*

I will discuss applications of Igusa integrals to quantitative arithmetic geometry.

Takehiko Yasuda

*The McKay correspondence and mass formulas for local Galois representations*

The McKay correspondence, which has been extensively studied in algebraic geometry for decades, relates invariants of quotient singularities (or resolutions of them) with more algebraic, in particular, representation-theoretic invariants. In this talk, I will overview my recent works on the McKay correspondence over the integer ring of a local field, parts of which are joint works with Melanie Matchett Wood and Fabio Tonini. This version of the McKay correspondence gives a new geometric interpretation of mass formulas for local Galois representations, which was previously studied only in the number-theoretic context by people including Krasner, Serre and Bhargava, and predicts motivic analogues of mass formulas.