1. Observe that $0 \leq \sin^2 x \leq 1$ on $[0, \pi/2]$, and therefore $0 \leq \frac{\sin^2 x}{x^{1/4}} \leq \frac{1}{x^{1/4}}$ on $(0, \pi/2]$. Because

$$\int_0^{\pi/2} x^{-1/4} dx = \lim_{c \to 0} \int_c^{\pi/2} x^{-1/4} dx = \lim_{c \to 0} \frac{4}{3} (\frac{\pi}{2})^{3/4} - \frac{4}{3} c^{3/4} < \infty$$

by the comparison theorem, the original integral converges as well.

2. There is a factorization $x^2 - x - 2 = (x - 2)(x + 1)$. Because on $(2, 3]$, $x + 1 \leq 4$, we have also $\frac{1}{x+1} \geq \frac{1}{4}$, and therefore $0 \leq \frac{1}{4(x-2)} \leq \frac{1}{x^2-x-2}$. But

$$\int_2^3 \frac{1}{4(x-2)} dx = \frac{1}{4} \lim_{c \to 2^+} (\ln(1) - \ln (c - 2)) \to \infty$$

by the comparison theorem, the original integral diverges.

2. Here we have $x^2 - 4x + 4 = (x - 2)^2$. We can therefore shift the integral to a more familiar form using a substitution $u = x - 2$. Now the integral becomes

$$\int_{-1}^{\infty} \frac{1}{u^2} du = \int_{-1}^{0} \frac{1}{u^2} du + \int_{0}^{1} \frac{1}{u^2} du + \int_{1}^{\infty} \frac{1}{u^2} du$$

We know the first two terms do not converge, therefore the integral diverges.

3. Again, using a substitution $u = x - 1$ we get the integral to a more familiar form:

$$\int_{-1}^{3} u^{-1/3} du = \int_{-1}^{0} u^{-1/3} du + \int_{0}^{3} u^{-1/3} du$$

Both of which we know to converge, thus the integral converges.

3. $a_n = (-1)^{n+1} \cdot 2n$

4. $a_n = (-1)^{n+1} \cdot 4^{2-n}$

5. $a_1 = 1, a_2 = 3$ are the initial terms. Inductively, for $n \geq 2$, $a_n = a_{n-1} + a_{n-2}$
1. Repeat L'Hopital's rule a number of times to see that this converges.

\[
\lim_{n \to \infty} \frac{n^3}{2n^3 - n} = \lim_{n \to \infty} \frac{3n^2}{6n^2 - 1} = \lim_{n \to \infty} \frac{6n}{12n} = \lim_{n \to \infty} \frac{6}{12} = \frac{1}{2}
\]

2. The sequence diverges. To see why, observe \( a_n = 1 + 3n \), which clearly goes to infinity as \( n \) approaches infinity.

3. This limit can be computed directly using some algebra.

\[
\lim_{n \to \infty} (e^{3n+4})^{1/n} = \lim_{n \to \infty} e^{4/n} \cdot e^3 = e^0 \cdot e^3 = e^3
\]

In particular, the sequence converges.

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1. Observe that \(-\pi \leq \arctan n \leq \pi\), hence \(-\frac{\pi}{n} \leq \frac{\arctan n}{n} \leq \frac{\pi}{n}\). Since \(\{-\frac{\pi}{n}\}\) and \(\{\frac{\pi}{n}\}\) both converge to 0, by the squeeze theorem, so does \(\{a_n\}\).

2. Since \(-1 \leq \sin n/2 \leq 1\), apply squeeze theorem again with bounds \(\{-1/n\}\) and \(\{1/n\}\) to see that \(\{a_n\}\) also converges to 0.

3. Simplifying notation we have \(a_n = (\frac{1}{(2n+1)2x})^{1/3}\). But because \(f(n) = a_n\) for \(f(x) = (\frac{1}{2x(2x+1)})^{1/3}\), and \(f(x) \to 0\) as \(x \to \infty\), \(a_n\) converges to 0 as well.