We prove by induction on \( n \). First notice that since \( a_1 = 1 \leq a_2 = 5/2 \leq 4 \), the sequence is increasing and bounded by 4 at \( n = 1 \). This fulfills the base case.

Now assume \( a_n \leq a_{n+1} \) and \( a_n \leq 4 \), then \( a_{n+1} = (4 + a_n)/2 \leq (4 + a_{n+1})/2 = a_{n+2} \), and \( a_{n+1} = (4 + a_n)/2 \leq (4 + 4)/2 = 4 \). This proves the induction step.

Since the sequence is increasing and bounded from above, by monotone converge theorem, the sequence converges to a limit, say \( \lim_{n \to \infty} a_n = L \). Then \( L \) satisfies \( L = (4 + L)/2 \). Solving for \( L \) gives \( L = 4 \), hence the sequence converges to 4.

This is a geometric series with \( a = 4/3 \) and \( r = 1/3 \). Since \( |r| < 1 \), the series converges to \( a/(1 - r) = 2 \).

Since \( \lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{n^2}{n^2 + 1} = 1 \), the series diverges by the divergence test.

This is geometric with \( a = 3/x \) and \( r = 3/x \). This converges when and only when \( |3/x| < 1 \), i.e., \( 3 < |x| \). If so, the limit is \( a/(1 - r) = 3/(x - 3) \).

By the integral test, it suffices to determine if \( \int_1^\infty \frac{1}{x \ln x} \, dx \) converges. But using \( u = \ln x \), we have

\[
\int \frac{1}{x \ln x} \, dx = \int \frac{1}{u} \, du = \ln u = \ln \ln x
\]

Thus the indefinite integral evaluates as \( \lim_{b \to \infty} \ln \ln b - 0 = \infty \), hence the series diverges.

Since \( 0 < n^4/2 \leq n^4 - 1 \) for \( n \geq 2 \), we have \( \frac{2n}{n^4} = \frac{2}{n^3} \geq \frac{n}{n^4 + 1} \geq 0 \). By the integral and p-test, \( \sum_{n=2}^{\infty} \frac{2}{n^3} \) converges, and therefore by comparison theorem for series, the original series converges as well.

Since \( 0 \leq \frac{3^{-n}}{1 + 3^{-n}} = \frac{3^{-n}}{1} \), by comparison theorem, it suffices to prove \( \sum_{n=1}^{\infty} 3^{-n} \) converges. But this is a geometric series with \( r = 1/3 \), hence it converges, and therefore so does the original series.